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This module was carefully examined and revised in accordance with the standards prescribed by the DepEd Region 4A and Curriculum and Learning Management Division CALABARZON. All parts and sections of the module are assured not to have violated any rules stated in the Intellectual Property Rights for learning standards.

The Editors
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First Quarter
Grade 11

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Pre-Calculus Grade 11
PIVOT IV-A Learner’s Material
Quarter 1
First Edition, 2020

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PIVOT 4A CALABARZON
Guide in Using PIVOT Learner’s Material

For the Parents/Guardian

This module aims to assist you, dear parents, guardians, or siblings of the learners, to understand how materials and activities are used in the new normal. It is designed to provide the information, activities, and new learning that learners need to work on.

Activities presented in this module are based on the Most Essential Learning Competencies (MELCs) for Pre-Calculus as prescribed by the Department of Education.

Further, this learning resource hopes to engage the learners in guided and independent learning activities at their own pace and time. Furthermore, this also aims to help learners acquire the needed 21st century skills while taking into consideration their needs and circumstances.

You are expected to assist the child in the tasks and ensure the learner’s mastery of the subject matter. Be reminded that learners have to answer all the activities in their own notebook.

For the Learners

The module is designed to suit your needs and interests using the IDEA instructional process. This will help you attain the prescribed grade-level knowledge, skills, attitude, and values at your own pace outside the normal classroom setting.

The module is composed of different types of activities that are arranged according to graduated levels of difficulty—from simple to complex. You are expected to answer all activities on separate sheets of paper and submit the outputs to your respective teachers on the time and date agreed upon.
## Parts of Pivot Learner’s Material

<table>
<thead>
<tr>
<th>Parts of the LM</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Introduction</strong></td>
<td>The teacher utilizes appropriate strategies in presenting the MELC and desired learning outcomes for the day or week, purpose of the lesson, core content and relevant samples. This allows teachers to maximize learners awareness of their own knowledge as regards content and skills required for the lesson.</td>
</tr>
<tr>
<td><strong>What I need to know</strong></td>
<td></td>
</tr>
<tr>
<td><strong>What is new</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Development</strong></td>
<td>The teacher presents activities, tasks, contents of value and interest to the learners. This shall expose the learners on what he/she knew, what he/she does not know and what he/she wanted to know and learn. Most of the activities and tasks must simply and directly revolved around the concepts to develop and master the skills or the MELC.</td>
</tr>
<tr>
<td><strong>What I know</strong></td>
<td></td>
</tr>
<tr>
<td><strong>What is in</strong></td>
<td></td>
</tr>
<tr>
<td><strong>What is it</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Engagement</strong></td>
<td>The teacher allows the learners to be engaged in various tasks and opportunities in building their KSA’s to meaningfully connect their learnings after doing the tasks in the D. This part exposes the learner to real life situations/tasks that shall ignite his/her interests to meet the expectation, make their performance satisfactory or produce a product or performance which lead him/her to understand fully the skills and concepts.</td>
</tr>
<tr>
<td><strong>What is more</strong></td>
<td></td>
</tr>
<tr>
<td><strong>What I can do</strong></td>
<td></td>
</tr>
<tr>
<td><strong>What else I can do</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Assimilation</strong></td>
<td>The teacher brings the learners to a process where they shall demonstrate ideas, interpretation, mindset or values and create pieces of information that will form part of their knowledge in reflecting, relating or using it effectively in any situation or context. This part encourages learners in creating conceptual structures giving them the avenue to integrate new and old learnings.</td>
</tr>
<tr>
<td><strong>What I have learned</strong></td>
<td></td>
</tr>
<tr>
<td><strong>What I can achieve</strong></td>
<td></td>
</tr>
</tbody>
</table>
Illustrating Different Types of Conic Sections

Lesson

After going through this module, you are expected to: 1.) Illustrate types of conic sections and degenerate cases; 2.) Define a circle; 3.) Determine the standard form and general form of the equation of the circle. 4.) Graph a circle in a rectangular coordinate system; and 5.) Recognize the importance of circle into real-life situation.

Learning Task 1: Use the figure below in answering each question about a circle.

1. What is the name of the circle?
2. What segment represents the radius and diameter of the circle?
3. How is radius related to the diameter of the circle?
4. Are there any tangent line in the figure? If yes, name the tangent line.
5. How is tangent line related to radius of the circle?

Learning Task 2: In determining the type of conic sections that can be generated in the activity, student will start imagining that he is trying to connect the dots of the two ice cream cones in which student can form a double napped cone.

Guide Questions

1. If you will cut the double napped cone using a plane figure horizontally, which of the following types of conic sections will be formed?

2. When the (tilted) plane intersects only one cone to form a bounded curve, which of the following types of conic sections will be formed?

3. When the plane intersects only one cone to form an unbounded curve, which of the following types of conic sections will be formed?

4. When the plane (not necessarily vertical) intersects both cones to form two unbounded curves, which of the following types of conic sections will be formed?
Discussion

Figure at the left shows the vivid illustration of the activity. It implies that when the plane figure cut the double napped cone horizontally then it will generate or form a **CIRCLE**. When the plane figure is tilted and cut only one cone to form a bounded curve then it generates an **ELLIPSE** while a **PARABOLA** is generated when it forms an unbounded curve. And when the plane figure cut the double napped cone not necessarily vertical to form two unbounded curves then it generates a **HYPERBOLA**. (Source: Learner Materials in Pre-Calculus)

There are other ways for a plane and the cones to intersect, to form what are referred to as degenerate conics: a point, a line, and two lines

**CIRCLE**

A circle is the locus of all points in the plane having the same fixed positive distance, called the radius, from a fixed point, called the center.

Equation of a circle in standard form.

An equation of a circle whose center is at \((h,k)\) and radius is \(r > 0\) is: \((x - h)^2 + (y - k)^2 = r^2\). It implies that when the radius is less than or equal to 0 therefore the equation is not a circle.

(1) center at the origin, radius 4

**Solution:** Since origin has an ordered pair of \((0, 0)\) therefore \(h = 0\) and \(k = 0\). By substituting the given values to the given formula, it will be \((x - 0)^2 + (y - 0)^2 = 4^2\). By simplifying, any number that you subtract from 0 the answer is itself therefore the final answer is \(x^2 + y^2 = 16\). In graphing a circle, plot the center \((0,0)\) in a rectangular coordinate system then count 4 times (radius) upward, downward, left and right then connect the dots by forming a circle as shown on the figure at the right side.

(2) center \((-4,3)\), radius \(\sqrt{7}\)

**Solution:** Since center is at \((-4, 3)\) therefore \(h = -4\) and \(k = 3\). By substituting the given values to the given formula, it will be \((x - (-4))^2 + (y - 3)^2 = (\sqrt{7})^2\)
By simplifying this \(-4\) the answer is positive 4 and \((\sqrt{7})^2\) will result to positive 7 therefore the final answer is \((x + 4)^2 + (y - 3)^2 = 7\). In graphing a circle, plot the center \((-4, 3)\) in a rectangular coordinate system then count \(\sqrt{7} \approx 2.65\) times (radius) upward, downward, left and right then connect the dots by forming a circle as shown on the figure at the right side.

(3) center \((5, -6)\), tangent to the y-axis

**Solution:** Since center is at \((5, -6)\) therefore \(h = 5\) and \(k = -6\). By substituting the given values to the given formula, it will be \((x - 5)^2 + (y - (-6))^2 = 5^2\) - note that if the circle is tangent to y-axis then the radius is equal to the value of \(h\) while if the circle is tangent to x-axis then the radius is equal to the value of \(k\). By simplifying this \(-6\) the answer is positive 6 therefore the final answer is \((x - 5)^2 + (y + 6)^2 = 25\). In graphing a circle, plot the center \((5, -6)\) in a rectangular coordinate system then count 5 times (radius) upward, downward, left and right then connect the dots by forming a circle as shown.

**General Form of the Equation of the Circle**

If the equation of a circle is given in the general form \(Ax^2 + Ay^2 + Cx + Dy + E = 0\), \(A \neq 0\), or \(x^2 + y^2 + Cx + Dy + E = 0\), we can determine the standard form by completing the square in both variables.

**Illustrative Examples**

(1) \(x^2 + y^2 -6x - 7 = 0\)

**Solution:**

\[
x^2 - 6x + y^2 = 7
\]
\[
(x^2 - 6x + 9) + y^2 = 7 + 9
\]
\[
(x - 3)^2 + y^2 = 16
\]
\[
C: (3, 0); r = 4
\]

By inspection, \(h = 3\), \(k = 0\) and \(r = \sqrt{16} = 4\)

(2) \(x^2 + y^2 -14x + 2y + 14 = 0\)

**Solution:**

\[
x^2 - 14x + y^2 + 2y = -14
\]
\[
(x^2 - 14x + 49) + (y^2 + 2y + 1) = -14 + 49 + 1
\]
\[
(x - 7)^2 + (y + 1)^2 = 36
\]
\[
C: (7, -1); r = 6
\]

By inspection, \(h = 7\), \(k = -1\) and \(r = 6\)
Determine the general form of the equation of a circle.

(1) center at the origin, radius 2

**Solution:**

\[(x-0)^2 + (y-0)^2 = 2^2\]

Substitute the values of h, k and r to standard form

\[x^2 + y^2 = 4\] Simplify \(x - 0 = x, y - 0 = y\) and \(2^2 = 4\)

\[x^2 + y^2 - 4 = 0\] Addition Property of Equality (APE)

(2) center \((-1,2),\) radius \(\sqrt{3}\)

**Solution:**

\[(x-(-1))^2 + (y-2)^2 = (\sqrt{3})^2\]

Substitute the values of h, k and r to standard form

\[(x + 1)^2 + (y - 2)^2 = 3\] Simplify \(-(-1) = 1\) and \((\sqrt{3})^2 = 3\)

\[x^2 + 2x + 1 + (y^2 - 4y + 4) - 3 = 0\] Apply square of binomial and APE

\[x^2 + y^2 + 2x - 4y + 2 = 0\] Combine like terms and follow the format of general form.

---

**Learning Task 3:** Determine and graph the standard form of the equation of the circle.

1: Center is at the origin and radius is 7
2: Center is at C \((2, -5)\) and radius is 3
3: Has a diameter whose endpoints are the points A \((3,4)\) and B \((-3,12)\)

**Learning Task 4: MATCH TO SOLVE!**

A German automobile manufacturer that designs, engineers, produces, markets and distributes luxury vehicles.

“____ ____ ____ ____”

1 2 3 4

**Direction:** Match column A with column B, by determining the standard and general equation of a circle.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. (x^2 + y^2 - 8x + 2y + 13 = 0)</td>
<td>I. ((x + 5)^2 + (y + 1)^2 = 4)</td>
</tr>
<tr>
<td>2. (x^2 + y^2 - 2x + 2y - 2 = 0)</td>
<td>U. ((x - 1)^2 + (y + 1)^2 = 4)</td>
</tr>
<tr>
<td>3. (x^2 + y^2 + 4x + 2y + 1 = 0)</td>
<td>D. ((x + 2)^2 + (y + 1)^2 = 4)</td>
</tr>
<tr>
<td>4. (x^2 + y^2 + 10x + 2y + 22 = 0)</td>
<td>A. ((x - 4)^2 + (y + 1)^2 = 4)</td>
</tr>
</tbody>
</table>

Source: Carla Casipis’s Module in Pre-Calculus—Division of Cavite Province
**Learning Task 5:** Transform the standard equation of a circle to general form and vice versa. Identify its radius and graph the circle.

<table>
<thead>
<tr>
<th>General Form of the Equation of a Circle</th>
<th>Radius</th>
<th>Standard Form of the Equation of a Circle</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^2 + y^2 - 4 = 0$</td>
<td>1.</td>
<td>2.</td>
</tr>
<tr>
<td></td>
<td>3.</td>
<td>$x^2 + y^2 = 144$</td>
</tr>
<tr>
<td>$x^2 + y^2 - 225 = 0$</td>
<td>5.</td>
<td>6.</td>
</tr>
<tr>
<td></td>
<td>7.</td>
<td>$x^2 + y^2 = 72$</td>
</tr>
<tr>
<td>$x^2 + y^2 - 6 = 0$</td>
<td>9.</td>
<td>10.</td>
</tr>
</tbody>
</table>

Source: Carla Casipis’s Module in Pre-Calculus—Division of Cavite Province
Learning Task 6: In this module, you learned two lessons that focused on standard form of the equation of a circle whose center is at the origin and \((h, k)\). You also experienced to apply what you have learned into real-life situation. Your task is to complete the reflection activity below.

I learned that

I realized that

I can apply what I have learned in

Learning Task 7: Do the indicated task individually. Apply the concepts you gained in this module in doing this performance task.

1. Take a photo of any circular object inside your house.
2. Trace that photo in a rectangular cartesian plane with 1-centimeter distance from each number.
3. Place the center of the photo in the origin of the rectangular Cartesian plane and measure the radius of the circle.
4. Think of a creative design in labeling the circular object in rectangular cartesian plane.
5. Determine the standard form and general form of the equation of the circular object.

Rubrics

<table>
<thead>
<tr>
<th>Category</th>
<th>Excellent</th>
<th>Very Satisfactory</th>
<th>Satisfactory</th>
<th>Needs Improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Content- Accuracy (20)</td>
<td>100% of the solutions are correct. (20)</td>
<td>80–99% of the solutions are correct (17)</td>
<td>60–79% of the solutions are correct (14)</td>
<td>Below 60% of the solutions are correct (11)</td>
</tr>
<tr>
<td>Presentation of Output (15)</td>
<td>Output is exceptionally attractive in terms of design, layout and neatness (15)</td>
<td>Output is attractive in terms of design, layout and neatness (13)</td>
<td>Output is acceptably attractive though it may be a bit messy (11)</td>
<td>Output is distractingly messy and not attractive (9)</td>
</tr>
<tr>
<td>Mathematical Content/ Reasoning (15)</td>
<td>Complete understanding of the mathematical concepts is evident in the presentation. (15)</td>
<td>Substantial understanding of the mathematical concepts is applied. (13)</td>
<td>Partial understanding of the mathematical concepts is applied. (11)</td>
<td>Limited understanding of the mathematical concepts is applied. (9)</td>
</tr>
</tbody>
</table>

Source: Carla Casipis’s Module in Pre-Calculus—Division of Cavite Province
Defining Parabola & Determining Its Standard Equation Form

Lesson

After going through this module, you are expected to: 1.) Define a parabola; 2.) Determine the standard form and general form of the equation of the parabola; 3.) Identify parts of a parabola given specific conditions; 4.) Graph a parabola in a rectangular coordinate system; and 5.) Recognize the importance of parabola into real-life situation.

Learning Task 1: Find the distance of any two points in a Cartesian plane.

1. A (2, 4) and B (-4, 4)
2. C (2, 4) and D (-4, -3)
3. E (1,3) and F (7, 11)
4. G (3, 3) and H (-3, 7)
5. I (7, 1) and J (-6, 5)

Guide Questions:
1. When are you going to use the distance formula?
2. Cite some real-life examples where you can apply the distance formula.
3. How did you find the distance of any two points in a Cartesian plane?

Learning Task 2: Derive the formula of the standard form of the equation of the parabola which opens upward. Apply the distance formula since based on illustration $\overline{PF}$ is equidistant to $\overline{PD}$.

Guide Questions:
1. What can you say about a parabola?
2. Where is the vertex located?
3. How did you derive with the standard form of the equation of the parabola which opens upward?
Parabola is locus of points such that the distance from a point to a focus is equal to the distance from the same point and the directrix. In deriving the standard form of the equation of the parabola, distance formula will be utilized. Based on the illustration, \( FP = PD \). Point F has coordinates \((0, p)\) therefore \(x_1 = 0\) and \(x_2 = p\) while point P has coordinates \((x, y)\) therefore \(x_2 = x\) and \(y_2 = y\). To find the distance between F and P, it will be \( \sqrt{(x - 0)^2 + (y - p)^2} \). Likewise, point D has coordinates \((x, -p)\) therefore \(x_1 = x\) and \(y_1 = -p\). To find the distance between P and D, it will be \( \sqrt{(x - x)^2 + (y - (-p))^2} \).

Since \( FP = PD \) therefore

\[
\sqrt{(x - 0)^2 + (y - p)^2} = \sqrt{(x - x)^2 + (y - (-p))^2}
\]

\(x^2 + (y - p)^2 = 0^2 + (y + p)^2\)  Squared both sides therefore square root will be cancelled.

\(x^2 + y^2 - 2py + p^2 = y^2 + 2py + p^2\)  Use square of binomial in simplifying \((y-p)^2\) and \((y+p)^2\).

\(x^2 = 2py + 2py\)  By combining like terms, \(y^2\) and \(p^2\) will be cancelled.

\(x^2 = 4py\)  Simplify \(2py + 2py = 4py\).

Therefore, the standard form of the equation of the parabola opens upward whose vertex is at the origin is \(x^2 = 4yp\). Here are other formulas of the parabola.

<table>
<thead>
<tr>
<th>Opening</th>
<th>Vertex at the Origin</th>
<th>Vertex at ((h, k))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upward</td>
<td>(x^2 = 4py)</td>
<td>((x-h)^2 = 4p(y - k))</td>
</tr>
<tr>
<td>Downward</td>
<td>(x^2 = -4py)</td>
<td>((x - h)^2 = -4p(y - k))</td>
</tr>
<tr>
<td>Right</td>
<td>(y^2 = 4px)</td>
<td>((y - k)^2 = 4p (x - h))</td>
</tr>
<tr>
<td>Left</td>
<td>(y^2 = -4px)</td>
<td>((y - k)^2 = -4p (x - h))</td>
</tr>
</tbody>
</table>

After deriving the standard form of the equation of the parabola, let us now discuss the parts of the parabola and the pattern that we can use in a specific

<table>
<thead>
<tr>
<th>Parts</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertex</td>
<td>sharpest turn point of the parabola (represented by V).</td>
</tr>
<tr>
<td>Focus</td>
<td>a point which is used to determine or define the parabola (represented by F). The distance of focus to the vertex is always determined by p.</td>
</tr>
<tr>
<td>Latus Rectum</td>
<td>a line passing through the focus, perpendicular to the axis of symmetry, and it has two end points. The distance of endpoints of latus rectum is always determined by 4p.</td>
</tr>
<tr>
<td>Axis of Symmetry</td>
<td>a line that divides the parabola in half.</td>
</tr>
<tr>
<td>Directrix</td>
<td>a line perpendicular to axis of symmetry (represented by D).</td>
</tr>
</tbody>
</table>
Patterns in Finding Parts of a Parabola

<table>
<thead>
<tr>
<th>PARABOLA OPENS UPWARD</th>
<th>Parts</th>
<th>Vertex at the Origin</th>
<th>Vertex at (h, k)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Focus</td>
<td>(0, p)</td>
<td>(h, k + p)</td>
<td></td>
</tr>
<tr>
<td>Endpoints of the Latus Rectum</td>
<td>$L_1: (-2p, p)$ &amp; $L_2: (2p, p)$</td>
<td>$L_1: (h-2p, k+p)$ &amp; $L_2: (h+2p, k+p)$</td>
<td></td>
</tr>
<tr>
<td>Equation of the Directrix</td>
<td>$y = -p$</td>
<td>$y = k-p$</td>
<td></td>
</tr>
<tr>
<td>Equation of Axis of Symmetry</td>
<td>$x = 0$</td>
<td>$x = h$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>PARABOLA OPENS DOWNWARD</th>
<th>Parts</th>
<th>Vertex at the Origin</th>
<th>Vertex at (h, k)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Focus</td>
<td>(0, -p)</td>
<td>(h, k - p)</td>
<td></td>
</tr>
<tr>
<td>Endpoints of the Latus Rectum</td>
<td>$L_1: (-2p, -p)$ &amp; $L_2: (2p, -p)$</td>
<td>$L_1: (h-2p, k-p)$ &amp; $L_2: (h+2p, k-p)$</td>
<td></td>
</tr>
<tr>
<td>Equation of the Directrix</td>
<td>$y = p$</td>
<td>$y = k - p$</td>
<td></td>
</tr>
<tr>
<td>Equation of Axis of Symmetry</td>
<td>$x = 0$</td>
<td>$x = h$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>PARABOLA OPENS TO THE RIGHT</th>
<th>Parts</th>
<th>Vertex at the Origin</th>
<th>Vertex at (h, k)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Focus</td>
<td>(p, 0)</td>
<td>(h+ p, k)</td>
<td></td>
</tr>
<tr>
<td>Endpoints of the Latus Rectum</td>
<td>$L_1: (p, -2p)$ &amp; $L_2: (p, 2p)$</td>
<td>$L_1: (h+p, k-2p)$ &amp; $L_2: (h+p, k+2p)$</td>
<td></td>
</tr>
<tr>
<td>Equation of the Directrix</td>
<td>$x = -p$</td>
<td>$x = h - p$</td>
<td></td>
</tr>
<tr>
<td>Equation of Axis of Symmetry</td>
<td>$y = 0$</td>
<td>$y = k$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>PARABOLA OPENS TO THE LEFT</th>
<th>Parts</th>
<th>Vertex at the Origin</th>
<th>Vertex at (h, k)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Focus</td>
<td>(-p, 0)</td>
<td>(h- p, k)</td>
<td></td>
</tr>
<tr>
<td>Endpoints of the Latus Rectum</td>
<td>$L_1: (-p, -2p)$ &amp; $L_2: (-p, 2p)$</td>
<td>$L_1: (h-p, k-2p)$ &amp; $L_2: (h-p, k+2p)$</td>
<td></td>
</tr>
<tr>
<td>Equation of the Directrix</td>
<td>$x = p$</td>
<td>$x = h + p$</td>
<td></td>
</tr>
<tr>
<td>Equation of Axis of Symmetry</td>
<td>$y = 0$</td>
<td>$y = k$</td>
<td></td>
</tr>
</tbody>
</table>

**Illustrative Examples**

I. Vertex at the Origin

Direction: Identify the parts of the given standard form or general form of the equation of the parabola and specific conditions.

1. $y^2 = 28x$

**Solution**

Opening: Right – since it follows $y^2 = 4px$

By observation, $p = 7$ since $28x = 4px$ (divide both sides by $4x$, you can get the value of $p$).

Since the value of $p = 7$, therefore by substituting this to the given pattern we can identify the parts of a parabola.

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Focus</th>
<th>Endpoints of Latus Rectum</th>
<th>Equation of Directrix</th>
<th>Equation of Axis of Symmetry</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 0)</td>
<td>(7, 0)</td>
<td>(7, -14) &amp; (7, 14)</td>
<td>$x = -7$</td>
<td>$y = 0$</td>
</tr>
</tbody>
</table>
Standard Form of the Equation of the Parabola: \( y^2 = 28x \)
General Form of the Equation of the Parabola: \( y^2 - 28x = 0 \)
Graph: See figure at the left

2. The length of the latus rectum is 12 and the parabola opens to the left.

Solution:
Since the length of the latus rectum is 12 therefore by definition \( p = 3 \) (since length of the latus rectum is equal to 4p therefore it will be 4p = 12 then divide both sides by 4, the answer is \( p = 3 \)). By substituting the value of \( p \) to the given pattern whose parabola opens to the left, we can arrive with the parts of a parabola.

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Focus</th>
<th>Endpoints of Latus Rectum</th>
<th>Equation of Directrix</th>
<th>Equation of Axis of Symmetry</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 0)</td>
<td>(-3, 0)</td>
<td>(-3, -6) &amp; (-3, 6)</td>
<td>( x = 3 )</td>
<td>( y = 0 )</td>
</tr>
</tbody>
</table>

Standard Form of the Equation of the Parabola: \( y^2 = -12x \)
General Form of the Equation of the Parabola: \( y^2 + 12x = 0 \)
Graph: see figure at the left

II. Vertex at \((h, k)\)
1. \( y^2 - 12y - 4x + 28 = 0 \)

Solution
Transform the general form of the equation of the parabola to its standard form.
\[
y^2 - 12y = 4x - 28
\]
\[
(y^2 - 12y + 36) = 4x - 28 + 36
\]
\[
(y - 6)^2 = 4(x + 2)
\]
Completing the square for \( y^2 - 12y \)
Factor the right side using perfect square trinomial and the left side using common monomial factor
Opening: to the right – since it follows \((y - k)^2 = 4p(x - h)\)
By observation, \( p = 1 \) since \( 4 = 4p \) (divide both sides by 4, you can get the value of \( p \)). Moreover, the value of \( h \) and \( k \) are \( h = -2 \) (since \( h \) is always the number beside \( x \)) and \( k = 6 \) (since \( k \) is always the number beside \( y \)).
Since the value of \( p = 1 \), \( h = -2 \) and \( k = 6 \), therefore by substituting this to the given pattern we can identify the parts of a parabola.

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Focus</th>
<th>Endpoints of Latus Rectum</th>
<th>Equation of Directrix</th>
<th>Equation of Axis of Symmetry</th>
</tr>
</thead>
<tbody>
<tr>
<td>((h, k))</td>
<td>((h + p, k))</td>
<td>((h+1, 6))</td>
<td>( x = h - p )</td>
<td>( y = k )</td>
</tr>
<tr>
<td>((-2, 6))</td>
<td>((-2+1, 6))</td>
<td>((-1, 6))</td>
<td>( x = -2 -1 )</td>
<td>( y = 6 )</td>
</tr>
</tbody>
</table>
Standard Form of the Equation of the Parabola: \((y - 6)^2 = 4(x + 2)\)

General Form of the Equation of the Parabola: \(y^2 - 12y - 4x + 28 = 0\)

Graph: see figure at the left

2. A parabola with focus at (2,5) and directrix \(y = -1\).

**Solution:**
Plot the focus- F (2, 5) and draw the line of the directrix \(y = -1\) to the Cartesian plane. Since the distance from the vertex to focus is \(p\) and vertex to directrix is \(-p\) therefore to locate the vertex, it is just the midpoint of the focus and directrix. The distance from focus to the directrix is 6 units. Divide this by 2 therefore \(p = 3\). To locate the vertex, just count 3 units downward from the focus. The vertex is located at (2, 2) and the parabola opens upward.
Since the value of \(p = 3\), \(h = 2\) and \(k = 2\), therefore by substituting this to the given pattern we can identify the parts of a parabola.

<table>
<thead>
<tr>
<th>Vertex ((h, k))</th>
<th>Focus ((h, k+p))</th>
<th>Endpoints of Latus Rectum (L_1: (h-2p, k+p)), (L_1: (2-6, 2+3)), (L_1: (-4, 5)), (L_2: (h+2p, k+p)), (L_2: (2+6, 2+3))</th>
<th>Equation of Directrix (y = k-p), (y = 2 - 3) (y = -1)</th>
<th>Equation of Axis of Symmetry (x = h), (x = 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2, 2)</td>
<td>(2, 2+3)</td>
<td>(2, 5)</td>
<td>(x = 2)</td>
<td></td>
</tr>
</tbody>
</table>

Standard Form of the Equation of the Parabola:
\((x-h)^2 = 4p(y-k) \rightarrow (x - 2)^2 = 12(y - 2)\)

General Form of the Equation of the Parabola:
\(x^2 - 4x + 4 = 12y - 24\)
\(x^2 - 4x - 12y + 28 = 0\)
Graph: see figure at the left

**Learning Task 3:** For you to fully master this module, transform the following standard form of equations to general form and vice-versa.

1. \(x^2 - 2x - 8y + 9 = 0\)
2. \((x-5)^2 = -8 (y + 1)\)
3. \(x^2 - 6x - 2y + 5 = 0\)
4. \(y^2 - 2x - 4y + 8 = 0\)
5. \((y + 9)^2 = 5 (x-17)\)
Learning Task 4: Fill in the table by identifying and sketching the parts of the given equation of a parabola

<table>
<thead>
<tr>
<th>Standard Form</th>
<th>(4x^2 - 128y = 0)</th>
<th>(y^2 - 10y - 40x + 25 = 0)</th>
<th>(x^2 - 18x + 36y - 207 = 0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Opening</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Value of (p)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vertex</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Focus</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Endpoints of Latus Rectum</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equation of the Directrix</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equation of the axis of symmetry</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Graph</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Learning Task 5: Identify and sketch the parts of a parabola by analyzing the given condition.

1. Given the focus at \((0,1)\), and vertex at the origin.
2. A parabola has vertex at \(V (6, -4)\) and focus at \(F (2, -4)\).
3. The focus has coordinates \((4, 0)\) with vertex at the origin.
4. The parabola with vertex at \(V (-3, -2)\) and directrix \(y = 3\).
5. A parabola has its vertex at \((-1, 5)\) and \(x = -5\) as directrix.
Learning Task 6: In this module, you learned two lessons that focused on standard form of the equation of a parabola whose center is at the origin and (h, k). You also experienced to apply what you have learned into real-life situation. Your task is to complete the reflection activity below.

I learned that
__________________________________________________________________________

I realized that
__________________________________________________________________________

I can apply what I have learned in
__________________________________________________________________________

Learning Task 7: Creative Parabolic Object
1. Think of a realistic parabolic object.
2. Draw the object in a rectangular coordinate system creatively.
3. You have the freedom to select on where the vertex will be located.
4. Identify the parts of a parabolic object
5. Determine the general and standard form of the equation of a parabolic object.

Rubrics

<table>
<thead>
<tr>
<th>Category</th>
<th>Excellent</th>
<th>Very Satisfactory</th>
<th>Satisfactory</th>
<th>Needs Improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Content-Accuracy (20)</td>
<td>100% of the solutions are correct (20)</td>
<td>80-99% of the solutions are correct (17)</td>
<td>60-79% of the solutions are correct (14)</td>
<td>Below 60% of the solutions are correct (11)</td>
</tr>
<tr>
<td>Presentation of Output (15)</td>
<td>Output is exceptionally attractive in terms of design, layout and neatness (15)</td>
<td>Output is attractive in terms of design, layout and neatness (13)</td>
<td>Output is acceptably attractive though it may be a bit messy (11)</td>
<td>Output is distracting-ly messy and not attractive (9)</td>
</tr>
<tr>
<td>Mathematical Content/ Reasoning (15)</td>
<td>Complete understanding of the mathematical concepts is evident in the presentation. (15)</td>
<td>Substantial understanding of the mathematical concepts is applied. (13)</td>
<td>Partial understanding of the mathematical concepts is applied. (11)</td>
<td>Limited understanding of the mathematical concepts is applied. (9)</td>
</tr>
</tbody>
</table>
Defining Ellipse and Determining Its Standard Equation Form

Lesson

After going through this module, you are expected to: 1.) Define an ellipse; 2.) Determine the standard form and general form of the equation of an ellipse; 3.) Identify parts of an ellipse given specific conditions; 4.) Graph an ellipse in a rectangular coordinate system; and 5.) Recognize the importance of ellipse into real-life situation.

Learning Task 1: Find the unknown values using the concept of Pythagorean Theorem.

1. \( a = 5 \quad b = 4 \quad c = ? \)
2. \( a = 24 \quad b = ? \quad c = 25 \)
3. \( a = ? \quad b = 15 \quad c = 17 \)
4. \( a = 10 \quad b = 6 \quad c = ? \)
5. \( a = 12 \quad b = ? \quad c = 15 \)

Guide Questions:
1. When are you going to use the Pythagorean theorem?
2. Cite some real-life examples where you can apply the Pythagorean theorem.
3. How did you find the unknown values using Pythagorean theorem?

Learning Task 2: Using the figure below in finding the distance of \( J_1G, J_1B, J_2G, J_2B, J_3G, J_3B, J_4G, J_4B, J_5G, J_5B \) and answer the given guide questions.

Guide Questions

1. What is the sum of the distances of \( J_1G & J_1B, J_2G & J_2B, J_3G & J_3B, J_4G & J_4B, \) and \( J_5G & J_5B \)?
2. What did you notice about the sum of the distances of the given pair of segments?
3. On what part of an ellipse does the sum of the distances of the pair of segments represent?

Source: Renz Pureza’s Module in Pre-Calculus—Division of Cavite Province
When the plane figure is tilted and cut only one cone to form a bounded curve then it generates an **ELLIPSE**. Based on the illustration, \( \frac{PF_1}{PF_2} + \frac{PF_2}{PF_1} = 2c \). It implies that the distance of segments from the the foci to any points on the ellipse is equal to the length of the major axis.

<table>
<thead>
<tr>
<th>Orientation</th>
<th>Vertex at the Origin</th>
<th>Vertex at (h, k)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal</td>
<td>( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 )</td>
<td>( \frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 )</td>
</tr>
<tr>
<td>Vertical</td>
<td>( \frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 )</td>
<td>( \frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1 )</td>
</tr>
</tbody>
</table>

**Note:** We can easily determine the orientation by comparing the denominators of the two fractions, if the higher denominator is on \( x^2 \) then the orientation is horizontal and if the higher denominator is on \( y^2 \) then the orientation is vertical.

### Parts of an Ellipse

**Orientation – HORIZONTAL**

<table>
<thead>
<tr>
<th>Parts</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Center C</td>
<td>This can be at the origin or ((h, k)). This is middle inner most part of an ellipse.</td>
</tr>
<tr>
<td>Vertices ((V_1 &amp; V_2))</td>
<td>The vertices are the parts on the ellipse, collinear with center and foci. Each vertex is a unit/s away from the center. The length of the endpoints of the vertices is called the major axis.</td>
</tr>
<tr>
<td>Co-Vertices ((W_1 &amp; W_2))</td>
<td>Each co-vertex is ( b ) unit/s away from the center. The length of the endpoints of the co-vertices is called the minor axis.</td>
</tr>
<tr>
<td>Foci ((F_1 &amp; F_2))</td>
<td>Each focus is ( c ) unit/s away from the center.</td>
</tr>
</tbody>
</table>

**Patterns in Finding Parts of an Ellipse**

- **Center (C):** \((0, 0)\)
- **Vertices:** \(V_1:(a, 0) \& V_2:(-a, 0)\)
- **Co-Vertices:** \(W_1:(0, b) \& W_2:(0, -b)\)
- **Foci:** \(F_1:(c, 0) \& F_2:(-c, 0)\)

**Orientation – VERTICAL**

<table>
<thead>
<tr>
<th>Parts</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Center C</td>
<td>This can be at the origin or ((h, k)). This is middle inner most part of an ellipse.</td>
</tr>
<tr>
<td>Vertices ((V_1 &amp; V_2))</td>
<td>The vertices are the parts on the ellipse, collinear with center and foci. Each vertex is a unit/s away from the center. The length of the endpoints of the vertices is called the major axis.</td>
</tr>
<tr>
<td>Co-Vertices ((W_1 &amp; W_2))</td>
<td>Each co-vertex is ( b ) unit/s away from the center. The length of the endpoints of the co-vertices is called the minor axis.</td>
</tr>
<tr>
<td>Foci ((F_1 &amp; F_2))</td>
<td>Each focus is ( c ) unit/s away from the center.</td>
</tr>
</tbody>
</table>

### Orientation – HORIZONTAL

- **Center:** \((0, 0)\)
- **Vertices:** \(V_1:(a, 0) \& V_2:(-a, 0)\)
- **Co-Vertices:** \(W_1:(0, b) \& W_2:(0, -b)\)
- **Foci:** \(F_1:(c, 0) \& F_2:(-c, 0)\)
### Illustrative Examples

#### Center at the Origin

**Direction:** Determine the standard form of the equation of an ellipse and identify its parts. After identifying, plot the coordinates of its parts and graph the standard form of the equation of an ellipse.

1. **$289x^2 + 64y^2 - 18496 = 0$**

**Solution:** First, transform the general form of the equation of an ellipse into standard form.

\[
289x^2 + 64y^2 - 18496 = 0 \quad \text{Write the general form of the equation of an ellipse.}
\]

\[
289x^2 + 64y^2 = 18496
\]

\[
\frac{x^2}{64} + \frac{y^2}{289} = 1
\]

Therefore, the standard form of the equation of an ellipse is

\[
\frac{x^2}{64} + \frac{y^2}{289} = 1
\]

Second, determine the orientation of the equation of an ellipse. Since the higher denominator is 289 which is below $y^2$ therefore the orientation is vertical. Third, identify the values of $a$, $b$ and $c$.

\[
a^2 = 289 \rightarrow a = \sqrt{289} \rightarrow a = 17
\]

\[
b^2 = 64 \rightarrow b = \sqrt{64} \rightarrow b = 8
\]

To find $c$, use the formula $c^2 = a^2 - b^2$.

\[
c^2 = a^2 - b^2
\]

\[
c^2 = 289 - 64
\]

\[
c^2 = 225 \rightarrow c = \sqrt{225} \rightarrow c = 15
\]

Since the orientation is vertical, center is at the origin and the values of $a$, $b$ and $c$ are 17, 8, and 15 consecutively therefore by substituting this to the given pattern we can identify the parts of an ellipse.
2. Find the parts of an ellipse whose center at the origin, vertical major axis of length 10 and minor axis of 6.

**Solution:** Since the length of the major axis (2a) is 10 therefore $a = 5$. Likewise, the length of the minor axis (2b) is 6 therefore $b = 3$.

To find $c$, use the formula $c^2 = a^2 - b^2$

$$c^2 = 25 - 9$$

$$c^2 = 16 \rightarrow c = \sqrt{16} \rightarrow c = 4$$

Since the orientation is vertical, center is at the origin and the values of $a$, $b$ and $c$ are 5, 3, and 4 consecutively therefore by substituting this to the given pattern we can identify the parts of an ellipse.

---

Standard Form of the Equation of an Ellipse:

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

Use the formula for standard form of the equation of an ellipse oriented vertically.

$$\frac{x^2}{3^2} + \frac{y^2}{5^2} = 1$$

Substitute the values of $a$ and $b$.

$$\frac{x^2}{3^2} + \frac{y^2}{5^2} = 1$$

Simplify $3^2 = 9$ and $5^2 = 25$. 
General Form of the Equation of an Ellipse:
\[ \frac{x^2}{9} + \frac{y^2}{25} = 1 \]

Write the standard form of the equation of an ellipse.

\[ 25x^2 + 9y^2 = 225 \]

Multiply the equation by the LCD (225).

\[ 25x^2 + 9y^2 - 225 = 0 \]

Use Addition Property of Equality. (Add both sides of the equation by -225.

II. Center at the \((h, k)\)

Direction: Identify the parts of the given specific conditions, standard form and general form of the equation of an ellipse.

1. \[ 144x^2 + 100y^2 - 1152x - 400y - 11696 = 0 \]

   **Solution:** First, transform the general form of the equation of an ellipse into standard form.

   \[ 144x^2 + 100y^2 - 1152x - 400y - 11696 = 0 \] Write the general form of the equation of an ellipse

   \[ 144x^2 - 1152x + 100y^2 - 400y = 11696 \] Combine like terms

   \[ 144(x^2 - 8x) + 100(y^2 - 4y) = 11696 \]

   Factor like terms using common monomial factoring

   \[ 144(x^2 - 8x + 16) + 100(y^2 - 4y + 4) = 11696 + 144(16) + 100(4) \]

   Use completing the squares for \((x^2 - 8x)\) and \((y^2 - 4y)\)

   \[ 144(x - 4)^2 + 100(y - 2)^2 = 14400 \]

   Factor the expressions in the parenthesis

   \[ \frac{(x - 4)^2}{100} + \frac{(y - 2)^2}{144} = 1 \]

   Multiply both sides by \(\frac{1}{14400}\)

   Second, determine the orientation of the equation of an ellipse. Since the higher denominator is 144 which is below \(y^2\) therefore the orientation is vertical.

   Third, identify the values of \(a, b\) and \(c\).

   \[ a^2 = 144 \rightarrow a = \sqrt{144} \rightarrow a = 12 \]

   \[ b^2 = 100 \rightarrow b = \sqrt{100} \rightarrow b = 10 \]

   To find \(c\), use the formula \(c^2 = a^2 - b^2\).

   \[ c^2 = a^2 - b^2 \]

   \[ c^2 = 144 - 100 \]

   \[ c^2 = 44 \rightarrow c = \sqrt{44} \rightarrow c = 2\sqrt{11} \approx 6.63 \]
Since the orientation is vertical, center is at the (4, 2) and the values of a, b and c are 12, 10, and $2\sqrt{11} \approx 6.63$ consecutively therefore by substituting this to the given pattern we can identify the parts of an ellipse.

<table>
<thead>
<tr>
<th>Center</th>
<th>Vertices</th>
<th>Co-Vertices</th>
<th>Foci</th>
</tr>
</thead>
<tbody>
<tr>
<td>(4, 2)</td>
<td>V₁:(h, k+a)</td>
<td>W₁:(h+b, k)</td>
<td>F₁:(h, k+c) &amp;</td>
</tr>
<tr>
<td></td>
<td>V₂:(4, 2+12)</td>
<td>W₂:(4+10, 2)</td>
<td>F₁:(4, 2+ 2\sqrt{11})</td>
</tr>
<tr>
<td></td>
<td>V₁:(4, 14)</td>
<td>W₁:(14, 2)</td>
<td>F₂: (h, k-c)</td>
</tr>
<tr>
<td></td>
<td>V₂: (h, k-a)</td>
<td>W₂: (h-b, k)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>V₂: (4, 2-12)</td>
<td>W₂: (4-10, 2)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>V₂: (4, -10)</td>
<td>W₂: (-6, 2)</td>
<td>F₂: (4, -4.63)</td>
</tr>
</tbody>
</table>

2. Find the parts of an ellipse whose major axis has length of 50 and foci 7 units left and right of the center (-3, 4).

**Solution:** Since the length of the major axis (2a) is 50 therefore a = 25. Likewise, the foci have length 7 units left and right therefore c = 7. Since the foci are counted left and right of the center therefore the orientation is horizontal.

To find b, use the formula $b^2 = a^2 - c^2$.

$$b^2 = a^2 - c^2$$

$$b^2 = 625 - 49$$

$$b^2 = 576 \rightarrow b = \sqrt{576} \rightarrow b = 24$$

Since the orientation is horizontal, center is at the (-3, 4) and the values of a, b and c are 25, 24, and 7 consecutively therefore by substituting this to the given pattern we can identify the parts of an ellipse.

<table>
<thead>
<tr>
<th>Center</th>
<th>Vertices</th>
<th>Co-Vertices</th>
<th>Foci</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-3, 4)</td>
<td>V₁:(22, 4) &amp; V₂: (-28, 4)</td>
<td>W₁:(-3, 28) &amp; W₂: (-3, -20)</td>
<td>F₁:(4, 4) &amp; F₂: (-10, 4)</td>
</tr>
</tbody>
</table>
Standard Form of the Equation of an Ellipse:

\[
\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1
\]
Write the standard form of the equation of an ellipse

\[
\frac{(x - (-3))^2}{25^2} + \frac{(y - 4)^2}{24^2} = 1
\]
Substitute the values of h, k, a and b.

\[
\frac{(x + 3)^2}{625} + \frac{(y - 4)^2}{576} = 1
\]
Simplify the values with parenthesis and squared.

General Form of the Equation of an Ellipse:

\[
\frac{(x + 3)^2}{625} + \frac{(y - 4)^2}{576} = 1
\]
Write the standard form of the equation of an ellipse.

\[
576(x^2 + 6x + 9) + 625(y^2 - 8y + 16) = 3
\]
Multiply the equation by the LCD (360,000) and expand the square of binomial.

\[
576x^2 + 3456x + 5184 + 625y^2 - 5000y + 10000 - 360000 = 0
\]
Distribute 576 and 625 inside the parenthesis and add -360,000 to both sides of the equation.

\[
576x^2 + 625y^2 + 3456x - 5000y - 344816 = 0
\]
Use the pattern of general form of the equation of an ellipse in arranging the terms.

**Learning Task 3:** Identify the values of a, b and c, and the orientation of the standard form of the equation of an ellipse.

1. \[
\frac{x^2}{36} + \frac{y^2}{100} = 1
\]
2. \[
\frac{(x + 2)^2}{25} + \frac{(y + 3)^2}{21} = 1
\]
3. \[
\frac{(x + 2)^2}{16} + \frac{(y - 7)^2}{121} = 1
\]
Learning Task 4: Determine the standard form of the equation of an ellipse and identify its parts. After identifying, plot the coordinates of its parts and graph the equations in one rectangular coordinate system then determine what real-life picture is in the graph.

<table>
<thead>
<tr>
<th>General Form</th>
<th>$9x^2 + 81y^2 - 729 = 0$</th>
<th>$121x^2 + 16y^2 - 1936 = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Form</td>
<td></td>
<td></td>
</tr>
<tr>
<td>a</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Orientation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Center</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vertices</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Co-Vertices</td>
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<tr>
<td>Foci</td>
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</tbody>
</table>

Learning Task 5: To find the answer, find the parts of an ellipse given the standard general form of its equation. Write the corresponding letter on the box above the correct answer in the decoder.

<table>
<thead>
<tr>
<th>Letter</th>
<th>Parts</th>
<th>Letter</th>
<th>Parts</th>
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</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Center</td>
<td>L</td>
<td>Center</td>
<td>A</td>
<td>Center</td>
</tr>
<tr>
<td>O</td>
<td>Vertices</td>
<td>E</td>
<td>Vertices</td>
<td>S</td>
<td>Vertices</td>
</tr>
<tr>
<td>Y</td>
<td>Co-vertices</td>
<td>P</td>
<td>Co-vertices</td>
<td>N</td>
<td>Co-vertices</td>
</tr>
<tr>
<td>T</td>
<td>Foci</td>
<td>M</td>
<td>Foci</td>
<td>R</td>
<td>Foci</td>
</tr>
<tr>
<td>U</td>
<td>Standard Form of the Equation of an Ellipse Graph</td>
<td>H</td>
<td>Standard Form of the Equation of an Ellipse Graph</td>
<td>B</td>
<td>Standard Form of the Equation of an Ellipse Graph</td>
</tr>
</tbody>
</table>

$\frac{(x - 7)^2}{25} + \frac{(y + 2)^2}{16} = 1$  $4x^2 + y^2 - 8x + 6y + 9 = 0$  $81x^2 + 4y^2 - 162x + 16y - 227 = 0$

$(1, -3)$ $(7, -2)$ $(10, -2)$ $(4, -2)$ $x^2 + \frac{y^2}{9} = 1$ $(12, -2)$ $(4, -2)$ $(10, -2)$ $(2, -2)$ $(1, -2)$ $(4, -2)$ $(10, -2)$ $(1, -5)$ $(1, -2)$ $(4, -2)$ $(10, -2)$ $(1, -5)$ $(3, -2)$ $(4, -2)$ $(10, -2)$ $(4, -2)$
Learning Task 6: In this module, you learned two lessons that focused on standard form of the equation of an ellipse whose center is at the origin and \((h, k)\). You also experienced to apply what you have learned into real-life situation. Your task is to complete the reflection activity below.

I learned that

I realized that

I can apply what I have learned in

Learning Task 7: Do the indicated task individually. Apply the concepts you gained in this module in doing this performance task.

1. Capture 1 elliptical object that you can see inside your house.
2. Trace that in a rectangular cartesian plane with 1-centimeter distance from each number.
3. Locate the center of an elliptical object and place the center at the origin.
4. Be creative in designing an elliptical object in rectangular cartesian plane.
5. Determine the standard form of the equation of an elliptical object and identify its parts.

Rubrics

<table>
<thead>
<tr>
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<th>Satisfactory</th>
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</table>
Defining Hyperbola and Determining Its Standard Equation Form

Lesson

After going through this module, you are expected to: 1.) Define a hyperbola; 2.) Determine the standard form and general form of the equation of a hyperbola; 3.) Identify parts of a hyperbola given specific conditions; 4.) Graph a hyperbola in a rectangular coordinate system; and 5.) Recognize the importance of hyperbola into real-life situation.

Learning Task 1: Draw the line of the given linear equation in a rectangular coordinate system.

1. \( y = 4x \)
2. \( y = 2x + 5 \)
3. \( y = -2x - 5 \)
4. \( y = \frac{7}{2}x \)
5. \( y = -\frac{7}{2}x \)

Guide Questions:
1. How will you determine the slope of the given linear equation?
2. What is the first thing that you need to do in drawing a line pertaining to linear equation?
3. How will you draw a line pertaining to linear equation?

Learning Task 2: Derive the formula of the standard form of the equation of a hyperbola which is oriented horizontally. Apply the distance formula by applying \( PF_1 - PF_2 = 2a \) and take note of \( c^2 = a^2 + b^2 \).

Guide Questions:
1. What can you say about a hyperbola?
2. Where is the center located?
3. How did you derive a the standard form of the equation of a hyperbola which is oriented horizontally?
A hyperbola is a curve that occurs at the intersection of a cone and a plane. It is also the set of all points in the plane such that the difference of their distances from two fixed points (foci) is constant. In deriving the standard form of the equation of a hyperbola, distance formula will be utilized. Based on the illustration, \( \frac{PF_1}{PF_2} = 2a \). It implies that the difference of the distance of PF1 and PF2 is equal to the length of a transverse axis.

<table>
<thead>
<tr>
<th>Orientation</th>
<th>Vertex at the Origin</th>
<th>Vertex at (h, k)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal</td>
<td>( \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 )</td>
<td>( \frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1 )</td>
</tr>
<tr>
<td>Vertical</td>
<td>( \frac{y^2}{a^2} - \frac{x^2}{b^2} = 1 )</td>
<td>( \frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1 )</td>
</tr>
</tbody>
</table>

Note: We can easily determine the orientation by looking which of x and y is positive. If y is positive, then the orientation is vertical and if x is positive then it is horizontal.

Patterns in Finding Parts of an Ellipse

<table>
<thead>
<tr>
<th>Parts</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Center C</td>
<td>This can be at the origin or (h, k). This is middle inner most part of an ellipse.</td>
</tr>
<tr>
<td>Vertices (V_1 &amp; V_2)</td>
<td>Each vertex is a units from the center. Moreover, the transverse axis has length 2a.</td>
</tr>
<tr>
<td>Endpoints of Conjugate Axis (W_1 &amp; W_2)</td>
<td>Each endpoint of conjugate axis is b units from the center. Moreover, the length of conjugate axis is 2b.</td>
</tr>
<tr>
<td>Foci (F_1 &amp; F_2)</td>
<td>Each focus is c units from the center and collinear to the vertices.</td>
</tr>
<tr>
<td>Asymptotes</td>
<td>These are the two lines that intersect at the center.</td>
</tr>
</tbody>
</table>

Patterns in Finding Parts of an Ellipse

<table>
<thead>
<tr>
<th>ORIENTATION – HORIZONTAL</th>
<th>Vertex at the Origin</th>
<th>Vertex at (h, k)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Center</td>
<td>C: (0, 0)</td>
<td>C: (h, k)</td>
</tr>
<tr>
<td>Vertices</td>
<td>V_1: (a, 0) &amp; V_2: (-a, 0)</td>
<td>V_1: (h+a, k) &amp; V_2: (-h, k)</td>
</tr>
<tr>
<td>Endpoints of Conjugate Axis</td>
<td>W_1: (0, b) &amp; W_2: (0, -b)</td>
<td>W_1: (h+b, k) &amp; W_2: (h-b, k)</td>
</tr>
<tr>
<td>Foci</td>
<td>F_1: (c, 0) &amp; F_2: (-c, 0)</td>
<td>F_1: (h+c, k) &amp; F_2: (h-c, k)</td>
</tr>
<tr>
<td>Equation of Asymptotes</td>
<td>( y = \pm \frac{b}{a}x )</td>
<td>( y = k \pm \frac{b}{a}(x-h) )</td>
</tr>
</tbody>
</table>
### ORIENTATION – VERTICAL

<table>
<thead>
<tr>
<th>Parts</th>
<th>Vertex at the Origin</th>
<th>Vertex at ((h, k))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Center</td>
<td>(C: (0, 0))</td>
<td>(C: (h, k))</td>
</tr>
<tr>
<td>Vertices</td>
<td>(V_1: (0, a)) &amp; (V_2: (0, -a))</td>
<td>(V_1: (h, k+a)) &amp; (V_2: (h, k-a))</td>
</tr>
<tr>
<td>Endpoints of Conjugate Axis</td>
<td>(W_1: (b, 0)) &amp; (W_2: (-b, 0))</td>
<td>(W_1: (h+b, k)) &amp; (W_2: (h-b, k))</td>
</tr>
<tr>
<td>Foci</td>
<td>(F_1: (0, c)) &amp; (F_2: (0, -c))</td>
<td>(F_1: (h, k+c)) &amp; (F_2: (h, k-c))</td>
</tr>
<tr>
<td>Equation of Asymptotes</td>
<td>(y = \frac{a}{b}x)</td>
<td>(y = k \pm \frac{a}{b} (x - h))</td>
</tr>
</tbody>
</table>

**Note:** In hyperbola, we use \(c^2 = a^2 + b^2\)

### Illustrative Examples

I. Center at the Origin

**Direction:** Identify the parts of the given specific conditions, standard form and general form of the equation of an ellipse.

\[
\frac{x^2}{25} + \frac{y^2}{11} = 1
\]

**Solution:** First, determine the orientation of the equation of a hyperbola. Since \(x\) is positive therefore the orientation is horizontal. Second, identify the values of \(a\), \(b\) and \(c\).

\[
a^2 = 25 \rightarrow a = 5 \quad \text{and} \quad b^2 = 11 \rightarrow \sqrt{11}
\]

To find \(c\), use the formula \(c^2 = a^2 + b^2\)

\[
c^2 = a^2 + b^2 \rightarrow c^2 = 25 + 11 \rightarrow c^2 = 36 \rightarrow c = 6
\]

Since the orientation is horizontal, center is at the origin and the values of \(a\), \(b\) and \(c\) are 5, \(\sqrt{11}\) and 6 consecutively therefore by substituting this to the given pattern we can identify the parts of a hyperbola.

<table>
<thead>
<tr>
<th>Center</th>
<th>Vertices</th>
<th>Endpoints of Conjugate Axis</th>
<th>Foci</th>
<th>Equation of the Asymptotes</th>
</tr>
</thead>
<tbody>
<tr>
<td>((0, 0))</td>
<td>(V_1: (5,0)) &amp; (V_2: (-5,0))</td>
<td>(w_1: (0, \sqrt{11})) &amp; (w_2: (0, -\sqrt{11}))</td>
<td>(F_1: (6,0)) &amp; (F_2: (-6,0))</td>
<td>(y = \pm \frac{6}{5}x)</td>
</tr>
</tbody>
</table>

Standard Form of the Equation of an Ellipse:

\[
\frac{x^2}{25} + \frac{y^2}{11} = 1
\]

General Form of the Equation of an Ellipse:

\[
11x^2 - 25y^2 - 275 = 0
\]
2. Find the parts of a hyperbola oriented horizontally whose center at the origin, length of transverse axis is 6 and length of conjugate axis is $4\sqrt{10}$.

**Solution:** Since the length of transverse axis (2a) is 6 therefore $a = 3$. Likewise, the length of the conjugate axis (2b) is $4\sqrt{10}$ therefore $b = 2\sqrt{10}$.

To find $c$, use the formula $c^2 = a^2 + b^2$

$$
c^2 = a^2 + b^2 \Rightarrow c^2 = 3^2 + (2\sqrt{10})^2 = 9 + 40 = c^2 = 49 \Rightarrow c = 7
$$

Since the orientation is horizontal, center is at the origin and the values of $a$, $b$ and $c$ are $3$, $2\sqrt{10}$, and 7 consecutively therefore by substituting this to the given pattern we can identify the parts of a hyperbola.

<table>
<thead>
<tr>
<th>Center</th>
<th>Vertices</th>
<th>Endpoints of Conjugate Axis</th>
<th>Foci</th>
<th>Equation of the Asymptotes</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 0)</td>
<td>$V_1: (0, 3)$, $V_2: (0, -3)$</td>
<td>$W_1: (2\sqrt{10}, 0)$, $W_2: (-2\sqrt{10}, 0)$</td>
<td>$F_1: (0, 7)$, $F_2: (0, -7)$</td>
<td>$y = \pm \frac{2\sqrt{10}}{3}$</td>
</tr>
</tbody>
</table>

Standard Form of the Equation of an Ellipse: $\frac{x^2}{9} + \frac{y^2}{40} = 1$

General Form of the Equation of an Ellipse: $40x^2 - 9y^2 - 360 = 0$

II. Center at (h, k)

Direction: Identify the parts of the given specific conditions, standard form and general form of the equation of a hyperbola.

$$
\frac{(y - 5)^2}{100} + \frac{(x + 1)^2}{25} = 1
$$

**Solution:** First, determine the orientation of the equation of an ellipse. Since $y$ is positive therefore the orientation is vertical. Second, identify the values of $a$, $b$ and $c$.

$$
a^2 = 100 \Rightarrow a = 10 \quad b^2 = 25 \Rightarrow b = 5
$$

To find $c$, use the formula $c^2 = a^2 + b^2$

$$
c^2 = a^2 + b^2 \Rightarrow c^2 = 100 + 25 \Rightarrow c^2 = 125 \Rightarrow c = 5\sqrt{5} \approx 11.18
$$

Since the orientation is vertical, center is at the (-1, 5) and the values of $a$, $b$ and $c$ are 10, 5, and $5\sqrt{5} \approx 11.18$ consecutively therefore by substituting this to the given pattern we can identify the parts of a hyperbola.
### Hyperbola Properties and Equations

<table>
<thead>
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<th>Endpoints of Conjugate Axis</th>
<th>Foci</th>
<th>Equation of the Asymptotes</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-1, 5)</td>
<td>$V_1: (h, k+a)$</td>
<td>$W_1: (h+b, k)$</td>
<td>$F_1: (h, k+c)$</td>
<td>$y = k + \frac{a}{b} (x - h)$</td>
</tr>
<tr>
<td></td>
<td>$V_1: (-1, 5+10)$</td>
<td>$W_1: (-1+5, 5)$</td>
<td>$F_2: (h, k-c)$</td>
<td>$y = 5 \pm \frac{10}{5} (x + 1)$</td>
</tr>
<tr>
<td></td>
<td>$V_1: (-1, 15)$</td>
<td>$W_1: (-1, 5)$</td>
<td>$F_3: (-1, 5 - 5\sqrt{5})$</td>
<td>$y = 5+2(x+1)$</td>
</tr>
<tr>
<td></td>
<td>$V_2: (h, k-a)$</td>
<td>$W_2: (h-b, k)$</td>
<td></td>
<td>$y = 5+2x+2$</td>
</tr>
<tr>
<td></td>
<td>$V_2: (-1, 5-10)$</td>
<td>$W_2: (-1-5, 5)$</td>
<td></td>
<td>$y = 2x + 7$</td>
</tr>
<tr>
<td></td>
<td>$V_2: (-1, -5)$</td>
<td>$W_2: (-6, 5)$</td>
<td></td>
<td>$y = 5-2(x+1)$</td>
</tr>
<tr>
<td></td>
<td>$W_1: (h+b, k)$</td>
<td>$W_2: (h-b, k)$</td>
<td></td>
<td>$y = 5-2x-2$</td>
</tr>
</tbody>
</table>

**Standard Form of the Equation of an Ellipse:**

\[
\frac{(y-5)^2}{100} - \frac{(x+1)^2}{25} = 1
\]

**General Form of the Equation of an Ellipse:**

\[
\frac{(y-5)^2}{25} - \frac{(x+1)^2}{100} = 1
\]

2. Find the parts of a hyperbola whose transverse axis has length of $6\sqrt{7}$ and foci 12 units left and right of the center (0, 2).

**Solution:** Since the length of the transverse axis (2a) is $6\sqrt{7}$ therefore $a = 3\sqrt{7}$. Likewise, the foci have length 12 units left and right therefore $c = 12$. Since the foci are counted left and right of the center therefore the orientation is horizontal.

To find $b$, use the formula $b^2 = c^2 - a^2$

\[b^2 = c^2 + a^2 - b^2 = 12^2 - (3\sqrt{7})^2 \rightarrow b^2 = 144 - 63 \rightarrow b^2 = 81 \rightarrow b = 9\]

Since the orientation is horizontal, center is at the $(0, 2)$ and the values of $a$, $b$ and $c$ are $3\sqrt{7}$, 9, and 12 consecutively therefore by substituting this to the given pattern we can identify the parts of a hyperbola.
<table>
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</thead>
<tbody>
<tr>
<td>(0, 2)</td>
<td>(V_1: (h+a, k))</td>
<td>(W_1: (h, k+b))</td>
<td>(F_1: (h+c, k))</td>
<td>(y = k \pm \frac{b}{a}(x - h))</td>
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<tr>
<td></td>
<td>(V_1: (0 + 3\sqrt{7}, 2))</td>
<td>(W_1: (0, 2+9))</td>
<td>(F_1: (0+12, 2))</td>
<td>(y = 2 \pm \frac{9}{3\sqrt{7}}x)</td>
</tr>
<tr>
<td></td>
<td>(V_2: (3\sqrt{7}, 2))</td>
<td>(W_2: (0, 2-9))</td>
<td>(F_2: (0-12, 2))</td>
<td>(y = \frac{3\sqrt{7}}{7}x + 2)</td>
</tr>
<tr>
<td></td>
<td>(V_2: (0 - 3\sqrt{7}, 2))</td>
<td>(W_2: (0, -7))</td>
<td>(F_2: (-12, 2))</td>
<td>(y = -\frac{3\sqrt{7}}{7}x + 2)</td>
</tr>
</tbody>
</table>

Standard Form of the Equation of an Ellipse: \(\frac{x^2}{63} - \frac{(y - 2)^2}{81} = 1\)

General Form of the Equation of an Ellipse:
\[
\frac{x^2}{63} - \frac{(y - 2)^2}{81} = 1 \\
81x^2 - 63(y^2 - 4y + 4) = 5103 \\
81x^2 - 63y^2 + 252y - 252 - 5103 = 0 \\
81x^2 - 63y^2 + 252y - 5000y - 5355 = 0
\]

**Learning Task 3:** Identify the length of transverse axis and conjugate axis, and the orientation of the given standard form of the equation of a hyperbola.

1. \(\frac{y^2}{25} - \frac{x^2}{16} = 1\)  
2. \(\frac{y^2}{27} + \frac{x^2}{169} = 1\)  
3. \(\frac{(x + 3)^2}{49} + \frac{(y + 1)^2}{15} = 1\)

**Learning Task 4:** Identify parts, standard form and general form of the equation of a hyperbola given its specific condition.

1. Find the parts of a hyperbola whose center is located at (-6, -3), one of its foci is located at (-11, -3) and one of the endpoints of the conjugate axis is located at (-6, 0).

2. The center of a hyperbola is at (-4,1), one of the vertices is at (-4,3) and the length of the conjugate axis is 6 units.
**Learning Task 5:** To find the answer, find the parts of a hyperbola given the standard general form of its equation. Write the corresponding letter on the box above the correct answer in the decoder.

**Decode the Answer**

It is an hour-glass shape, which means that it has two hyperbolas one on each side.

\[
\frac{(x - 1)^2}{4} + \frac{(y - 1)^2}{12} = 1
\]

1. Orientation:

<table>
<thead>
<tr>
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<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>Center</td>
<td></td>
</tr>
<tr>
<td>K</td>
<td>Vertices</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>Endpoints of Conjugate Axis</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>Foci</td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>Equation of Asymptotes</td>
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Graph:

2. \[121y^2 - 4x^2 - 484 = 0\]

Orientation:

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</tr>
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<td>C</td>
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<td></td>
</tr>
</tbody>
</table>

Graph:
Learning task 6: After going through the different activities in this module, I am sure that you learned a lot. I want you to share with me your thoughts about our lesson, by completing the following statements:

I learned that ____________________________________
___________________________________________________.

I want to learn more about ______________________
___________________________________________________.

I can apply what I learned in _____________________
___________________________________________________.

Learning Task 7: Designing a Vase

I am sure that you are now equipped with skills and knowledge about hyperbola that could help you in designing a hyperbolic object.

I want you to create your own design a hyperbolic vase of the table. You will be sketching this in a rectangular coordinate system with 1 centimeter distance from each number. You will submit a sketch of the design with corresponding equation and parts of a hyperbola. Your output will be graded using the given rubric.

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Source: Haydee Mojica’s Module in Pre-Calculus—Division of Cavite Province
Solving Problems Involving Iconic Functions
Lesson

After going through this module, you are expected to: 1.) Transform general to standard form of the equation; 2.) Recognize the types of conics and its orientation given the standard and general form of the equation; 3.) Identify the type of degenerate cases given the standard and general form of the equation; 4.) Solve problems involving conic sections; 5.) Appreciate the significant value of conics into real-life situation.

**Learning Task 1:** Given the standard form of the equation. Identify the type of conics being presented.

1. \( \frac{x^2}{72} + \frac{y^2}{21} = 1 \)
2. \( (x - 1)^2 + (y + 3)^2 = 1 \)
3. \( \frac{(x - 7)^2}{2} - (y + 5)^2 = 1 \)
4. \( (y + 1)^2 = 20(x + 9) \)
5. \( \frac{x^2}{100} + \frac{y^2}{25} = 1 \)

Guide Questions:
1. What is your basis in identifying the types of conics given the standard form of the equation?
2. Based on our previous discussion, can you determine the orientation of given type of conics?

**Learning Task 2:** Transform the following general form of the equation into standard form and identify the type of conics, and the values of A and B on each example.

1. \( 11x^2 + 7y^2 + 22x - 126y + 501 = 0 \)
2. \( 2x^2 + 2y^2 - 20x - 4y - 20 = 0 \)
3. \( 2y^2 - 15x^2 - 40y - 90x + 35 = 0 \)
4. \( y^2 - 8y - 12x - 68 = 0 \)

Guide Questions:
1. How did you transform the general form to standard form of the given equation?
2. Did you see any pattern to easily recognize the type of conics given the general form of the equation?
3. What do you think will be the role of identifying the values of A and B given the general form of the equation in recognizing the type conics?
Recognize conic sections and degenerate cases

The relationship between coefficients $A$ and $B$ are significant in identifying the type of conics and its characteristics.

If $A = B \neq 0$, the equation represents a circle.
If $A \neq B$ and $AB > 0$, the equation represents an ellipse.
The orientation of an ellipse is horizontal when $A < B$ and vertical when $A > B$.
If $A = 0$ or $B = 0$, but not both, the equation represents a parabola.
The parabola opens a.) upward when $B = 0$ and $D < 0$, b.) downward when $B = 0$ and $D>0$, c.) to the right when $A = 0$ and $C < 0$ and d.) to the left when $A = 0$ and $C>0$
If $AB < 0$, the equation represents a hyperbola.
The orientation of a hyperbola is horizontal when $A> 0$ and vertical when $B > 0$.

In recognizing type of conics representing a standard form of the equation, it is essential to familiarize yourself with the following:

Circle: $(x-h)^2 + (y-k)^2 = r^2$

Ellipse: \[
\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \quad : \text{if } a^2 > b^2 \text{ then the orientation if horizontal and if } b^2 > a^2 \text{ then the orientation is vertical.}
\]

Parabola: If $(x-h)^2 = 4p(y-k)$ then the parabola opens upward. If $(x-h)^2 = -4p(y-k)$ then the parabola opens downward. If $(y-k)^2 = 4p(x-h)$ then the parabola opens to the right. If $(y-k)^2 = -4p(x-h)$ then the parabola opens to the left.

Hyperbola: \[
\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1 \quad \text{If then the orientation of hyperbola is horizontal.}
\]

In determining the types of degenerate cases, it is essential to familiarize yourself with the following:

If the left side in the standard form of the equation of a circle and ellipse are equal to 0 then it is a point while if it has negative value, then it is an empty set.

If the left side in the standard form of the equation of a hyperbola is equal to 0 then it forms two intersecting lines. (Source: Learners Materials in Pre-Calculus)

Illustrative Examples

I. Direction: Identify the type of conics and its orientation given the standard form of the equation.

<table>
<thead>
<tr>
<th>Standard Form of the Equation</th>
<th>Type of Conics</th>
<th>Orientation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{x^2}{15} + \frac{y^2}{121} = 1$</td>
<td>Ellipse</td>
<td>Vertical since $121 &gt; 15$</td>
</tr>
<tr>
<td>$\frac{(y-1)^2}{125} - (x + 1)^2 = 1$</td>
<td>Hyperbola</td>
<td>Vertical since $y$ comes first before $x$</td>
</tr>
<tr>
<td>$(x-17)^2 = -80(y+12)$</td>
<td>Parabola</td>
<td>Downward since it follows the formula $(x-h)^2 = -4p(y-k)$</td>
</tr>
</tbody>
</table>
II Direction: Identify the the values of A and B, type of conics and its orientation given the general form of the equation.

<table>
<thead>
<tr>
<th>General Form of the Equation</th>
<th>A</th>
<th>B</th>
<th>Type of Conics</th>
<th>Orientation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 5x² + 55x–12y + 450 = 0</td>
<td>5</td>
<td>0</td>
<td>Parabola</td>
<td>Upward since A&gt;0, B=0 and D&lt;0</td>
</tr>
<tr>
<td>2.-11x² + 20y² + 18x –15y + 25 = 0</td>
<td>-11</td>
<td>20</td>
<td>Hyperbola</td>
<td>Vertical since B&gt;A</td>
</tr>
<tr>
<td>3. 5x² + 5y² + 20x –30y + 100 = 0</td>
<td>5</td>
<td>5</td>
<td>Circle</td>
<td>None</td>
</tr>
<tr>
<td>4. 121x² + 16y² + 4x–128y + 292 = 0</td>
<td>121</td>
<td>16</td>
<td>Ellipse</td>
<td>Vertical since A&gt;B</td>
</tr>
</tbody>
</table>

III. Direction: Transform the given general form of the equation and identify the types of degenerate cases.
1. \( x^2 + 5x + y^2 - y + 7 = 0 \)

   **Solution:**
   \[
   \begin{align*}
   &x^2 + 5x + y^2 - y + 7 = 0 \\
   &\left(x^2 + 5x + \frac{25}{4}\right) + \left(y^2 - y + \frac{1}{4}\right) = -7 + \frac{25}{4} + \frac{1}{4} \\
   &\left(x + \frac{5}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{1}{4}
   \end{align*}
   \]

Since the left side of the standard form of a circle has a negative value therefore it is an **empty set**.

2. \( 36x^2+360x-64y^2-512y-124 = 0 \)

   **Solution:**
   \[
   \begin{align*}
   &36x^2+360x-64y^2-512y-124 = 0 \\
   &36(x^2 + 10x + 25) - 64(y^2 + 8y + 16) = 124 + 900 - 1024 \\
   &36(x+5)^2 - 64(y+4)^2 = 0
   \end{align*}
   \]

Since the left side of the standard form of a hyperbola has a 0 value therefore it

**Situational Problems Involving Conic Sections**

In solving situational problems involving conic sections, it is essential to follow these steps:
1. Identify the given.
2. Determine what is being asked.
3. Show solution

**Illustrative Examples**

1. Romans built their amphitheaters always in an ovoid shape, or in simple words, an ellipse. One of the amphitheaters in Rome is the **Colosseum**, known for being the largest of all the others, as well as the fact that it has been a part of Roman history. The Colosseum, from its exterior walls, is 189 meters long and 156 meters wide. It has a central arena (with the same elliptical shape) which is 87 meters long and 55 meters wide. Given that the Colosseum (and the arena) are both horizontally aligned, if the Colosseum’s center is found on the origin of the Cartesian plane (0,0), find the distance between the amphitheater’s foci and their same-side central arena’s vertices (i.e. distance between the focus with a positive x and the C.A. vertex with a positive x, and the same for negative).

*PIVOT 4A CALABARZON*
Identify the given values.
Major axis of the Colosseum = 189 meters; \( a_1 = 94.5 \) m; Vertices = (94.5, 0) and (-94.5, 0)
Minor axis of the Colosseum = 156 meters; \( b_1 = 78 \) m; Co-Vertices = (0, 78) and (0, -78)
Major axis of the central arena = 87 meters; \( a_2 = 43.5 \) m; Vertices = (43.5, 0) and (-43.5, 0)
Minor axis of the central arena = 55 meters; \( b_2 = 27.5 \) m; Co-Vertices = (0, 27.5) and (0, -27.5)

Determine what is being asked
Find the distance between the amphitheater’s foci and their same-side central arena’s vertices (i.e. distance between the focus with a positive x and the C.A. vertex with a positive x, and the same for negative).

Solution
The formula for the foci is given as \( c^2 = a^2 - b^2 \). To find the foci of the Colosseum, we need to substitute the given values to the formula.

\[
\begin{align*}
    c_1^2 &= a_1^2 - b_1^2 \\
    c_1^2 &= (94.5 \text{ m})^2 - (78 \text{ m})^2 \\
    c_1^2 &= 8930.25 \text{ m}^2 - 6084 \text{ m}^2 \\
    c_1^2 &= 2845.25 \text{ m}^2 \\
    c_1 &\approx 53.35 \text{ m}
\end{align*}
\]

Now, we know that the foci of the ellipse are situated at (52.35, 0) and (-52.35, 0) because the center of the ellipse is at the origin. From this we can use the distance formula between the foci and the vertices of the central arena, or we can simply subtract the two x values.

\[
d = \sqrt{(x_2 - x_1)^2 - (y_2 - y_1)^2} = \sqrt{\pm c_1 - \pm a_2}^2 - (0 - 0)^2
\]

\[
d = \sqrt{(52.35 - 43.5)^2}
\]

\[
d = 52.35 - 43.5 = 8.85 \text{ m}
\]

Final Answer
Therefore, the distance between the foci of the Colosseum and its central arena’s vertices is 8.85 m.

2. A student from Tanza National Comprehensive High School got curious about the circular logo just outside their school. He speculates that the logo has a diameter of 2 meters. He tells his math teacher that the equation of the circle can be \( 2x^2 + 2y^2 - 4x - 4y - 2 = 0 \) if 1 meter is treated as 1 unit. His teacher says that his constant is wrong. What should the constant be to make his answer right?
Given:
The diameter of the circle is 2 meters.
The equation of the circle can be expressed as \(2x^2 + 2y^2 + 4x + 4y - k = 0\) for some constant \(k\).

What is being asked?
What should the constant be to make his answer right?

Solution:
We can use the student’s equation and compress it into the standard form of the equation, also with the use of completing the square.

\[
x^2 + y^2 - 2x - 2y - \frac{k}{2} = 0
\]
\[
(x^2 - 2x) + (y^2 - 2y) = \frac{k}{2}
\]
\[
(x^2 - 2x + 1) + (y^2 - 2y + 1) = \frac{k}{2} + 2
\]
\[
(x + 1)^2 + (y + 1)^2 = \frac{k + 4}{2}
\]

Because 1 meter is treated as 1 unit, and the radius of the circle is 1 meter, \(k + 4\) then must be \(1^2\) equal to, or basically, 1.

\[
\frac{k + 4}{2} = 1
\]
\[
k + 4 = 2
\]
\[
k = -2
\]

Substituting back to the student’s equation, it should then be:

\[2x^2 + 2y^2 - 4x - 4y + 2 = 0\]

Final Answer:
Therefore, the constant should be (positive) 2.

3. As nighttime approached, a woman opened the lampshade in her room, and immediately noticed that the lamp cast vertical hyperbolic shadows on the wall. As she had nothing else to do, she decided to do some math. She found out that the center of the hyperbola was 3 units away from the vertices, and the foci were 2 units away from the vertices. If the center is situated at the origin, find the general form of the equation of these hyperbolic shadows, along with the lengths of its transverse and conjugate axes.

Given:
Center: (0,0); Vertices: (0,3) and (0, -3); Foci: (0,5) and (0, -5)

What is being asked?
If the center is situated at the origin, find the general form of the equation of these hyperbolic shadows, along with the lengths of its transverse and conjugate axes.
Solution:

The standard form of the equation of a vertical hyperbola with center at the origin is \( \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \). Since we already have \( a \) (distance from vertices to center) and \( c \) (distance from foci to center), we only need \( b \) (distance from co-vertices to center). To find this, we can use the formula \( c^2 = a^2 + b^2 \) and manipulate it to get a formula for finding \( b \).

\[
b^2 = c^2 - a^2
\]
\[
b^2 = 5^2 - 3^2 = 25 - 9 = 16
\]
\[
b = 4
\]

We can then substitute this to the equation.

\[
\frac{y^2}{3^2} - \frac{x^2}{4^2} = 1
\]
\[
\frac{y^2}{9} - \frac{x^2}{16} = 1
\]
\[
6y^2 - 9x^2 = 144
\]
\[
16y^2 - 9x^2 - 144 = 0
\]

The transverse axis in a hyperbola is the segment connecting the two vertices, therefore, the formula for this is \( 2a \). Easily, we get its length by substituting, and we get \( 6 \) units.

On the contrary, the conjugate axis is the segment connecting the co-vertices, which means the formula for this is \( 2b \). We then get \( 8 \) units as an answer.

Final Answer

The general form of the equation of the hyperbolic shadow is \( 16y^2 - 9x^2 - 144 = 0 \) with \( 6 \) units as length of transverse axis and \( 8 \) units as length of conjugate axis.

Learning Task 3: Identify the type of conics or degenerate cases of the following general form of the equation. Determine also the orientation if the equation represents a type of conics.

A. Circle  B. Parabola  C. Ellipse  D. Hyperbola  
E. Point  F. Empty Set  G. Two Intersecting Lines

1. \( x^2 + y^2 - 18y - 19 = 0 \)
2. \( x^2 - 10x - 48y + 244 = 0 \)
3. \( -5x^2 + 60x + 7y^2 + 84y + 72 = 0 \)
4. \( -144x^2 - 1152x + 25y^2 - 150y - 5679 = 0 \)
5. \( 9x^2 - 72x - 16y^2 - 128y - 256 = 0 \)
Learning Task 4: Identify the type of conics or degenerate cases of the following standard form of the equation. Determine also the orientation if the equation represents a type of conics.

A – Circle  B – Parabola  C. Ellipse  D. Hyperbola  E. Point  F. Empty Set  G. Two Intersecting Lines

1. 
2. 
3. 
4. 
5. 

Learning Task 5: Solve each situational problem involving conic sections.

1. A mathematician was shopping for furniture when he stumbles across the clocks area and finds different sizes of clocks. He then thinks what he’s finding in the clock that he would buy. He tells the shopkeeper that he would like a clock with a diameter that is irrational. The shopkeeper then gave him 3 clocks as a challenge. The first clock had this equation: ; the second had the ends of its diameter on the points (3,6) and (9,8); the last clock had an area of 144 square units. Which clock(s) would fit the mathematician’s request?

2. An engineer is working on making a diagram for the cooling tower he was going to construct in the future. Cooling towers can be seen as towers with inward-curving sides, or simply, hyperbola-looking sides. His daughter asks him how high the center of the hyperbolic sides of the tower was , and he answers that the center can be situated at (0,15) in the Cartesian plane. The engineer then says that the asymptotes of the hyperbola are the following: , and the area of its auxiliary rectangle is 168 square units. The daughter then finds the foci of the hyperbola. At what points on the Cartesian plane are these foci found on?

3. Suspension bridges are known to be normal around the world, especially in developed countries. These bridges usually have towers of some sort and cables, both of which have great importance in the integral stability of the bridge. A STEM Student makes some plans for his project on a suspension bridge model. He first tries to draw his model. The two towers are 12 meters high and 20 meters apart (10 meters away each from the origin in the center), and the cable he drew between and from the top of the two towers are of course, parabolic. The lowest point of the cable is 3 meters above the origin. He shows this model to his teacher, and the teacher asks him, “What is the vertical distance from the road to the cable at 5 meters away from the center?” What is the answer to the teacher’s question?

4. A certain celestial body is orbiting around a star somewhere in space. An astronomer tries to graph out the elliptical orbit of the celestial body using dimensional analysis and scaling. In his drawing, the celestial body is in the origin, and he finds out that the equation of the elliptical orbit is for some value of k. If his findings are true, find the location of the star (the center of the ellipse) in the Cartesian plane.
Learning Task 6: After going through the different activities in this module, I am sure that you learned a lot. I want you to share with me your thoughts about our lesson, by completing the following statements:

I have learned that ________________________________________
__________________________________________________________.

I realized that _____________________________________________
___________________________________________________________.

Learning Task 7: PHOTO CONIC SECTIONS: You will be working individually about the application of conic sections into real-life situation by capturing photos of different conic sections that you can see within the town. Also, you need to construct own situational problems that have in connection with each picture. The solution of each problem is also needed.

GOAL: You are task to capture photos of different conic sections that you can see within your town and construct own problems involving each picture.

ROLE: The students will act as photographer and problem maker.

AUDIENCE: The audience will be the Municipal Mayor.

SITUATION: The Municipal Mayor of your town, would like to know the ravishing and flabbergasting establishments at your town. One of his consultants, gave him an advice that the best pictures that they can present are those pictures that have in connection with conic sections. However, he is very problematic because the Provincial Governor asked him to present pictures so that they can be able to see the captions and the histories of each pictures.

PRODUCT: The photographer will give stunning presentations of pictures in a unique way with captions, problems and solutions.

STANDARDS:

<table>
<thead>
<tr>
<th>CRITERIA</th>
<th>RATING</th>
</tr>
</thead>
<tbody>
<tr>
<td>Presentation of the Output (15)</td>
<td>Output is presented in a clear and orderly manner. (15)</td>
</tr>
<tr>
<td>Applications of Mathematical Concepts (20)</td>
<td>Presented at least 8 pictures with problems involving conic sections. (20)</td>
</tr>
<tr>
<td>Mathematical Content/Reasoning (15)</td>
<td>Complete understanding of the mathematical concepts is evident in the presentation. (15)</td>
</tr>
</tbody>
</table>
After going through this module, you are expected to: 1.) Illustrate a series; 2.) Differentiate series from sequence; 3.) Apply the concepts of different types of sequences and series; and 4.) Realize the significant value of sequence and series in real-life situation.

**Learning Task 1: Let’s Recall Sequence**

1. What are the different types of sequences that you have learned in Mathematics 10?
2. What are the formulas that you still remember in sequences?
3. Cite some situational problems where you can apply sequences in daily lives.

**Learning Task 2: Analyze and solve the given situational problem by applying the concepts of different types of sequences and series.**

1. Mrs. Marilou Cabugo is a Biology teacher at Tanza National Comprehensive High School. One of the experiments she facilitated is doing experiment thru series of trials. They will be testing the invented mosquito repellant. The table below shows the result of each trial.

<table>
<thead>
<tr>
<th>Trials</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Result</td>
<td>8</td>
<td>40</td>
<td>200</td>
</tr>
</tbody>
</table>

Based on the result, how many mosquitoes will be killed using the mosquito repellant on the 6th trial.

2. Mr. Christian Espineli is a MAPEH teacher, one of his tasks to his student is to do a series of sit ups. Table below shows the number of sit-ups that each student needs to do.

<table>
<thead>
<tr>
<th>Level</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of Sit-ups needed</td>
<td>3</td>
<td>9</td>
<td>15</td>
</tr>
</tbody>
</table>

On what level can a student reach 45 sit-ups. If a student reached 45 sit-ups then how many total number of sit-ups does he did from level 1 up to that level?
Guide Questions
1. What is the difference of the first problem to the second?
2. How did you find the nth term of the given sequence in the first and second problem?
3. How did you find the total number of sit-ups that a student reached after the 8th level? What mathematical term did you apply in solving this problem?
4. What is the difference of sequence to a series?

Discussion
A sequence is a list of numbers (separated by commas), while a series is a sum of numbers (separated by “+” or “−” sign). The sequence with nth term \(a_n\) is usually denoted by \{\(a_n\)\}, and the associated series is given by \(S = a_1 + a_2 + a_3 + \cdots + a_n\). (Source: Learners’ Materials in Pre-Calculus)

The first to eight level on the second problem requires a number of trials represented by this \{3, 9, 15, 21, 27, 33, 39, 45\} and this illustrates a sequence. This will become a series if this will be represented as 3 + 9 + 15 + 21 + 27 + 33 + 39 + 45. Therefore, the associated series is 192.

Illustrative Examples
I. Direction: Complete the table by writing the first three terms of a defined sequence and give its associated series.

<table>
<thead>
<tr>
<th>Defined Sequence</th>
<th>Sequence</th>
<th>Series</th>
<th>Associated Series</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. {4 − 2n}</td>
<td>(A_1 = 4 - 2(1) = 2)</td>
<td>2 + 0 + -2</td>
<td>(S_3 = 0)</td>
</tr>
<tr>
<td></td>
<td>(A_2 = 4 - 2(2) = 0)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(A_3 = 4 - 2(3) = -2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. {3 − 4n + n^2}</td>
<td>(A_1 = 3 - 4(1) + 1^2 = 0)</td>
<td>0 + -1 + -5</td>
<td>(S_3 = -6)</td>
</tr>
<tr>
<td></td>
<td>(A_2 = 3 - 4(2) + 2^2 = -1)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(A_3 = 3 - 4(3) + 3^2 = -5)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. {(-2)^n}</td>
<td>(A_1 = (-2)^1 = -2)</td>
<td>-2 + 4 + -8</td>
<td>(S_3 = -6)</td>
</tr>
<tr>
<td></td>
<td>(A_2 = (-2)^2 = 4)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(A_3 = (-2)^3 = -8)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

II. Direction: Solve the given problem involving sequence and series.

<table>
<thead>
<tr>
<th></th>
<th>Sequence</th>
<th>Series</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arithmetic</td>
<td>(a_n = a_1 + (n - 1)d)</td>
<td>(S_n = \frac{n}{2} (2a_1 + (n - 1)d)) or (S_n = \frac{n}{2} (a_1 + a_n))</td>
</tr>
<tr>
<td>Geometric</td>
<td>(a_n = a_1r^{n-1})</td>
<td>(S_n = \frac{a_1(1-r^n)}{(1-r)})</td>
</tr>
</tbody>
</table>

Let’s recall first the formula of arithmetic and geometric sequences.

1. Find the sum of the first 15 terms of the arithmetic sequence -25, -20, -15, ...

Solution:
Given: \(a_1 = -25\) \hspace{1cm} n = 15 \hspace{1cm} d = 5
Write the formula of arithmetic series

\[ S_n = \frac{n}{2}[2a_1 + (n - 1)d] \]

Substitute the values of \(a_1, n\) and \(d\)

\[ S_n = \frac{15}{2}[2(-25) + (15 - 1)5] \]

Simplify

\[ S_n = \frac{15}{2} \]

\[ S_n = 150 \]

2. What is the sum of all positive integers less than 200 that are multiples of 11?

Solution:

Given: \(a_1 = 11\) \(n = ?\) \(d = 11\) \(a_n = 198\) (largest number multiple by 11 and less than 200)

<table>
<thead>
<tr>
<th>Finding n</th>
<th>Finding the sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a_n = a_1 + (n - 1)d)</td>
<td>(S_n = \frac{n}{2}[a_1 + a_n])</td>
</tr>
<tr>
<td>198 = 11 + (n - 1)11</td>
<td>(S_n = \frac{18}{2}[11 + 198])</td>
</tr>
<tr>
<td>198 = 11 + 11n - 11</td>
<td>= 9(209)</td>
</tr>
<tr>
<td>198 = 11n</td>
<td>= 1881</td>
</tr>
<tr>
<td>18 = n</td>
<td></td>
</tr>
</tbody>
</table>

3. The third term of a geometric sequence is \(-18\) and the sixth term is \(486\). Find the second term.

Given:

\[ a_3 = -18 \quad a_6 = 486 \]

\[ a_3 = a_1r^2 = -18 \quad a_6 = a_1r^5 \]

Eliminating \(a_1\) by division; \(\frac{a_6}{a_3} = r^3 = \frac{486}{-18}\). Thus, \(r = -3\).

Since \(a_3 = a_2r^2\), therefore \(-18 = a_2(-3)^2\). Thus \(a_2 = -2\).

Learning Task 3: Like or Unlike. The students are going to write LIKE if the given represents sequence and UNLIKE if the given represents series.

1. 1, 2, 4, 8, . . .
2. 2, 8, 16, 32, . . .
3. \(-1 + 1 - 1 + 1 - 1\)
4. \(1/2, 2/3, 3/4, 4/5, . . .\)
5. \(1 + 2 + 22 + 23 + 24\)
6. \(1 + 0.1 + 0.001 + 0.0001\)
7. \(5, 15, 25, 35, 45, 55\)
8. \(9 + 12 + 15 + 18 + 21\)
9. \(10 + 20 + 40 + 80 + 160\)
10. \(-1, 3, -9, 27, -81\)
Learning Task 4: Determine the first five terms of each defined sequence, and give their associated series.

1. \( \{2 + n^3\} \)
2. \( \{3n - 10 + 5n^2\} \)
3. \( \{(-3)^n - 10\} \)
4. \( \{1 + 2 + 3 + \cdots + n\} \)
5. \( \{2 - 3(n - 1)\} \)

Learning Task 5: Analyze and solve the given problem.

1. Find the sum of the first 150 counting numbers.
2. Find the sum of the first 50 odd natural numbers.
3. Find the sum of all the even integers from 12 to 864, inclusive.
4. List the first three terms of the arithmetic sequence if the 25th term is 35 and the 30th term is 5.
5. Find the 5th term of the arithmetic sequence whose 3rd term is 35 and whose 10th term is 77.

Learning Task 6: After going through the different activities in this module, I am sure that you learned a lot. I want you to share with me your thoughts about our lesson, by completing the following statements:

I have learned that ________________________________________
________________________________________________________________.

I realized that ____________________________________________
________________________________________________________________.

Learning Task 7: (1) Apply a real-life problem of a sequence and series (2) Construct this in a problem, and show solutions and answers on the given problem. (Use the rubrics below in grading you output)

<table>
<thead>
<tr>
<th>Category</th>
<th>Excellent</th>
<th>Very Satisfactory</th>
<th>Satisfactory</th>
<th>Needs Improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Content-Accuracy (20)</td>
<td>100% of the solutions are correct (20)</td>
<td>80-99% of the solutions are correct (17)</td>
<td>60-79% of the solutions are correct (14)</td>
<td>Below 60% of the solutions are correct (11)</td>
</tr>
<tr>
<td>Mathematical Content/ Reasoning (15)</td>
<td>Complete understanding of the mathematical concepts is evident in the presentation. (15)</td>
<td>Substantial understanding of the mathematical concepts is applied. (13)</td>
<td>Partial understanding of the mathematical concepts is applied. (11)</td>
<td>Limited understanding of the mathematical concepts is applied. (9)</td>
</tr>
</tbody>
</table>
Using Sigma Notation to Represent a Series

Lesson

After going through this module, you are expected to: 1.) Illustrate a sigma notation; 2.) Use sigma notation to represent a series; 3.) Apply the use of sigma notation in finding sums; and 4.) Recognize the significant value of sigma notation in real-life situation.

Learning Task 1: Find the nth term of the given sequence

For example: 2, 5, 8, 11, ....., the nth term of the given sequence can be represented as $A_n = 3n-1$.

1. 3, 4, 5, 6, 7, .......
2. 3, 5, 7, 9, 11, ....
3. 2, 4, 8, 16, 32, ....
4. -1, 1, -1, 1, -1, ....
5. $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$

Learning Task 2: Find the sum of the given problem.

1. Find the sum of the first five natural even numbers.

2. If the defined sequence is $4m$ then find the sum of the given sequence if $m = 3$ up to 7.

3. If the defined sequence is $3m - 2$ then find the sum of the given sequence if $m = 8$ up to 11.

Guide Questions

1. How did you find the sum of the given problem?

2. If you are not allowed to use arithmetic series, what other mathematical term can be used in finding the sum of the given problem?

3. How are you going to describe sigma notation?
Sigma Notation

Mathematicians use the sigma notation to denote a sum. The uppercase Greek letter Σ (sigma) is used to indicate a “sum.” The notation consists of several components or parts. The picture at the right can be read as “the summation of f of i from m up to n.

For instance, the problem is “find the sum of the first five natural even numbers”. To represent this problem using sigma notation, let’s start with writing the sequence which is 2, 4, 6, 8, 10. As we can observe, using our knowledge in finding nth term of a sequence, it can be represented as \( A_m = 2m \). Use this as basis in writing on the summand part and for the lower bound, it starts with \( m = 1 \) up to 5. Thus, the representation of this problem using sigma notation is

\[
\sum_{m=1}^{5} 2m
\]

Properties of Sigma Notation

1. \( \sum_{i=1}^{n} ka_i = k \sum_{i=1}^{n} a_i \)

For instance, the problem is “if the defined sequence is \( 4m \) then find the sum of the given sequence if \( m = 3 \) up to 7”.

\[
\sum_{m=3}^{7} 4m = 4 \sum_{m=3}^{7} m = 4(3 + 4 + 5 + 6 + 7) = 100
\]

2. \( \sum_{i=1}^{n} (a_i \pm b_i) = \sum_{i=1}^{n} a_i \pm \sum_{i=1}^{n} b_i \)

For instance, the question is “if the defined sequence is \( 3m - 2 \) then find the sum of the given sequence if \( m = 8 \) up to 11”.

\[
\sum_{m=8}^{11} 3m - 2 = \sum_{m=8}^{11} 3m - \sum_{m=8}^{11} 2
\]

\[
= [3(8) + 3(9) + 3(10) + 3(11)] - 2(4)
\]

\[
= [24 + 27 + 30 + 33] - 8
\]

\[
= 114 - 8
\]

\[
= 106
\]

Illustrative Examples

I. Direction: Expand each summation and simplify if possible.

1. \( \sum_{i=0}^{5} 2^i \)

Solution:

\[
\sum_{i=0}^{5} 2^i = 2^0 + 2^1 + 2^2 + 2^3 + 2^4 + 2^5 \quad \text{Substitute } i = 0 \text{ to } 5
\]

\[
= 1 + 2 + 4 + 8 + 16 + 32 \quad \text{Simplify numbers with exponent}
\]

= 63
2. \[ \sum_{i=4}^{6} 5i \]

Solution:
\[
\sum_{i=4}^{6} 5i = 5(4) + 5(5) + 5(6) + 5(7) + 5(8)
\]
Substitute \( i = 4 \) to 8
\[
= 20 + 25 + 30 + 35 + 40
\]
Simplify the numbers with parenthesis
\[
= 150
\]

3. \[ \sum_{i=1}^{4} (2i + 3) \]

Solution:
\[
\sum_{i=1}^{4} (2i + 3) = [2(2) + 3] + [2(3) + 3] + [2(4) + 3]
\]
Substitute \( i = 2 \) to 4
\[
= 7 + 9 + 11
\]
Simplify values in bracket
\[
= 27
\]

4. \[ \sum_{k=1}^{20} (2k-1) \]

Solution:
\[
\sum_{k=1}^{20} (2k-1) = \sum_{k=1}^{20} 2k - \sum_{k=1}^{20} 1
\]
\[
= \left[ \frac{20 \times (2 + 40)}{2} \right] - 20
\]
Use formula of arithmetic series instead of expanding up to 20\textsuperscript{th} term.
Simplify
\[
= 420 - 20
\]
\[
= 400
\]

II. Direction: Use sigma notation to represent a series.

1. \( 3 + 10 + 17 + 24 + 31 \)

Solution:
As we can notice, the given represents an arithmetic series. Let us identify the defined sequence of the given series.
This explanation is applicable for arithmetic sequence.

\[ d_n = x \] first term \hspace{1cm} The common difference is 7 and the first term is 3.
\[ 7n = x \] If \( n = 1 \) then we have \( 7(1) = x \). Thus \( x = -4 \).

So, the defined sequence is \( 7n - 4 \)

Since \( n = 1 \) satisfies the defined sequence and there are only 5 terms therefore, the sigma notation that can represent the given series is
\[
\sum_{n=1}^{5} 7n - 4
\]
2. \(a_2 + a_4 + a_6 + a_8 + \ldots + a_{20}\)
Solution: By observation, the subscript of the given variable is divisible by 2. Since it ends with 20 therefore it implies that there are 10 terms in the given series. Thus, the sigma notation that can represent the given series is
\[
\sum_{n=1}^{10} a_{2n}
\]

3. \(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} + \frac{1}{128}\)
Solution: By observation, the denominator has base of 2 with exponent from 0 to 7. Thus, the sigma notation that can represent the given series is
\[
\sum_{n=0}^{7} \frac{1}{2^n}
\]

**Learning Task 3:** Use the properties of sigma notation in simplifying the given problem.

1. \(\sum_{k=1}^{5} (2k)\)
2. \(\sum_{k=2}^{6} (3k - 5)\)
3. \(\sum_{k=0}^{4} (3k + 2)^2\)
4. \(\sum_{k=2}^{30} (2k - 1)\)
5. \(\sum_{k=1}^{4} \frac{k + 1}{2k}\)

**Learning Task 4:** Illustrate the series using the sigma notation then simplify.

1. \(5 + 10 + 15 + \ldots + 50\)
2. \(3 + 7 + 11 + \ldots + 39\)
3. \(9 + 27 + 81 + \ldots + 6561\)
4. \(1 + 2 + 4 + 8 + \ldots + 1024\)
5. \(\frac{1}{3} + \frac{4}{3} + \frac{7}{3} + \ldots + \frac{31}{3}\)
Learning Task 5: Analyze the given condition and calculate the sum using the properties of sigma notation.

1. Given that \( a_1 = 42 \) and \( \sum_{n=1}^{10} a_n = 2014 \), find the following:
   1.1. \( \sum_{n=1}^{10} 5a_n \)
   1.2. \( \sum_{n=1}^{10} (a_n - 2) \)

2. Given that \( a_1 = 3 \) and \( \sum_{n=2}^{25} a_n = 55 \), find the following:

\[
2.1. \sum_{n=1}^{25} 5a_n
\]

\[
2.2. \sum_{n=1}^{25} (2a_n + 3)
\]

Learning Task 6: After going through the different activities in this module, I am sure that you learned a lot. I want you to share with me your thoughts about our lesson, by completing the following statements:

I have learned that

I realized that
**Learning Task 7: IPON CHALLENGE**

This pandemic required us to budget and save money.

As days passed, we learned to maximize all the resources we have at home and save even the centavo of our money.

As to this, I challenge you!

Start saving your money now!

I will give you two options for your savings:

First option, in each preceding week you will add a fix amount. (Let us say each week, you will add 20 pesos)

Second option, in each preceding week you will double the amount.

Direction:

Make a calendar-like diagram, put also a date. It is your own choice of the day but make sure the intervals of the days are the same. Indicate also the amount you will save in each day.

At the end, let us see which option will make you save more money.

Illustrate the sigma notation in calculating your savings until end week of December.

<table>
<thead>
<tr>
<th></th>
<th>OCTOBER</th>
<th>NOVEMBER</th>
<th>DECEMBER</th>
<th>SUM</th>
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<tbody>
<tr>
<td></td>
<td>1ST</td>
<td>2ND</td>
<td>3RD</td>
<td>4TH</td>
</tr>
<tr>
<td>1st Option</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2nd Option</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

First option: _______________________________

Second option: _______________________________

Rubrics:

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Excellent</th>
<th>Very Satisfactory</th>
<th>Satisfactory</th>
<th>Needs Improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Content (10)</td>
<td>Complete parts are written. (10)</td>
<td>2-3 parts are missing. (8)</td>
<td>4-5 parts are missing. (5)</td>
<td>6-7 parts are missing. (4)</td>
</tr>
<tr>
<td>Presentation of Output (1.0)</td>
<td>Output is exceptionally attractive in terms of design, layout and neatness (10)</td>
<td>Output is attractive in terms of design, layout and neatness (8)</td>
<td>Output is acceptably attractive though it may be a bit messy (6)</td>
<td>Output is distractingly messy and not attractive (4)</td>
</tr>
<tr>
<td>Mathematical Content/Reasoning (1.5)</td>
<td>Complete understanding of the mathematical concepts is evident in the presentation. (15)</td>
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<td>Limited understanding of the mathematical concepts is applied. (9)</td>
</tr>
</tbody>
</table>

Source: Mariz N. Lansak’s Module in Pre-Calculus—Division of Cavite Province
Answer Key

**LESSON 1**

**Learning Task 1:**
1. Circle A
2. Radius: __________
3. Diameter: __________
4. Yes, line
5. Tangent line is perpendicular to the radius of the circle

**Learning Task 2:**
1. Circle
2. Ellipse
3. Parabola
4. Hyperbola

**Learning Task 3:**
1. $x^2 + y^2 = 49$
2. $(x-2)^2 + (y+5)^2 = 9$
3. $x^2 + (y-8)^2 = 25$

**Learning Task 4:**
1. A
2. U
3. D
4. I

**Learning Task 5:**
1. $x^2 + y^2 = 49$
2. $x^2 - 10x + 8y + 33 = 0$
3. $(x-3)^2 = 2(y+2)$
4. $(y-2)^2 = 2(x-2)$
5. $y^2 + 18y - 5x + 166 = 0$

**LESSON 2**

**Learning Task 1:**
1. 6 units
2. __________ units
3. 10 units
4. __________ units

**Learning Task 3:**
1. $(x-1)^2 = 8(y-1)$
2. $x^2 - 10x + 8y + 33 = 0$
3. $(x-3)^2 = 2(y+2)$
4. $(y-2)^2 = 2(x-2)$
5. $y^2 + 18y - 5x + 166 = 0$

**Learning Task 4:**
1. A
2. D
3. U
4. I

**Learning Task 5:**
1. A
2. D
3. U
4. I

**LESSON 3**

**Learning Task 1:**
1. Circle
2. Ellipse
3. Parabola
4. Hyperbola

**Learning Task 2:**
5. Tangent line is perpendicular to the radius of the circle

**Learning Task 3:**
1. Yes, line
2. Diameter: AB
3. Radius: AC, AD
4. Diameter: BD

**LESSON 4**
Learning Task # 5

<table>
<thead>
<tr>
<th>Orientation</th>
<th>Vertex</th>
<th>Focus</th>
<th>Latus Rectum</th>
<th>Directrix</th>
<th>Axis of Symmetry</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical</td>
<td>$(0, 0)$</td>
<td>$(0, 1)$</td>
<td>$L_1: (-2, 1)$ &amp; $L_2: (2, 1)$</td>
<td>$y = -1$</td>
<td>$x = 0$</td>
</tr>
<tr>
<td>Horizontal</td>
<td>$x^2 - 4y = 0$</td>
<td>$x^2 = 4y$</td>
<td>$L_1: (2, -4)$ &amp; $L_2: (2, 4)$</td>
<td>$y = -4$</td>
<td>$x = 10$</td>
</tr>
<tr>
<td>Vertical</td>
<td>$(0, 0)$</td>
<td>$(4, 0)$</td>
<td>$L_1: (4, 8)$ &amp; $L_2: (4, -8)$</td>
<td>$x = -4$</td>
<td>$y = 0$</td>
</tr>
<tr>
<td>Horizontal</td>
<td>$y^2 = 16x$</td>
<td>$y^2 - 16x = 0$</td>
<td>$L_1: (2, -2)$ &amp; $L_2: (2, 7)$</td>
<td>$x = -3$</td>
<td>$y = 3$</td>
</tr>
<tr>
<td>Vertical</td>
<td>$(0, 0)$</td>
<td>$(3, 5)$</td>
<td>$L_1: (3, 13)$ &amp; $L_2: (3, -3)$</td>
<td>$y = 5$</td>
<td>$x = -5$</td>
</tr>
</tbody>
</table>

Learning Task # 3

<table>
<thead>
<tr>
<th>Orientation</th>
<th>Vertex</th>
<th>Focus</th>
<th>Latus Rectum</th>
<th>Directrix</th>
<th>Axis of Symmetry</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical</td>
<td>$(0, 0)$</td>
<td>$(0, 1)$</td>
<td>$L_1: (0, 1)$ &amp; $L_2: (0, -1)$</td>
<td>$y = 1$</td>
<td>$x = 0$</td>
</tr>
<tr>
<td>Horizontal</td>
<td>$x = 0$</td>
<td>$y = 0$</td>
<td>$L_1: (1, 0)$ &amp; $L_2: (-1, 0)$</td>
<td>$x = 1$</td>
<td>$y = 1$</td>
</tr>
<tr>
<td>Vertical</td>
<td>$(0, 0)$</td>
<td>$(0, 1)$</td>
<td>$L_1: (0, 1)$ &amp; $L_2: (0, -1)$</td>
<td>$y = 1$</td>
<td>$x = 0$</td>
</tr>
<tr>
<td>Horizontal</td>
<td>$x = 0$</td>
<td>$y = 0$</td>
<td>$L_1: (1, 0)$ &amp; $L_2: (-1, 0)$</td>
<td>$x = 1$</td>
<td>$y = 1$</td>
</tr>
</tbody>
</table>
### Learning Task # 4

#### Vertices

<table>
<thead>
<tr>
<th>SF</th>
<th>GF</th>
<th>( a )</th>
<th>( b )</th>
<th>( c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>(-2, -3) &amp; (-10, -3)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td>(-4, 3) &amp; (-2, -3)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### Equation of the Asymptotes

1. \( 9x^2 - 16y^2 + 108x - 96y + 36 = 0 \)
2. \( 9y^2 - 4x^2 - 32x - 18y - 91 = 0 \)

### Learning Task # 5

**Decoder:** KOBE PORT TOWER

**Lesson 4**

**Treatment**

**Lithotripsy**

**Learning Task # 3**

#### Vertical

- \( \frac{2}{3} \) \( \text{TA} \)
- \( \frac{10}{3} \) \( \text{CA} \)
- \( 8 \) \( \text{O} \)

#### Horizontal

- \( \frac{6}{5} \) \( \text{TA} \)
- \( 14 \) \( \text{CA} \)
- \( \frac{2}{5} \) \( \text{O} \)

- \( \frac{6}{5} \) \( \text{TA} \)
- \( \frac{10}{3} \) \( \text{CA} \)
- \( 8 \) \( \text{O} \)

- \( \frac{6}{5} \) \( \text{TA} \)
- \( \frac{10}{3} \) \( \text{CA} \)
- \( 8 \) \( \text{O} \)

### Learning Task # 1

**Picture: Airplane**

- P: \( (0, 0) \)
- V: \( (4, 0) \)
- W: \( (4, 6) \)
- A: \( (1, 1) \)

**Diagonal**

- \( 0 = 12x - 12y \)
- \( 0 = 3x + 3y \)

**2 Join Points**

- \( I = \frac{1}{x^2} + \frac{1}{x^2} \)
Learning Task # 1
1. Ellipse
2. Circle
3. Hyperbola
4. Parabola
5. Ellipse

Learning Task # 2
1. $A = 11$, $B = 7$, Ellipse
   - Vertical
2. $A = 2$, $B = 2$, Circle
   - None
3. $A = -15$, $B = 2$, Hyperbola
   - Vertical
4. $A = 0$, $B = 1$, Parabola
   - Right

Learning Task # 3
1. $A$ – Upward
2. $B$ – Vertical
3. $C$ – Vertical
4. D – Vertical
5. D – Horizontal

Learning Task # 4
1. E
2. B – Right
3. C – Vertical
4. D – Vertical
5. F

Learning Task # 5
1.
2.
3.
4.

Learning Task # 1
1. $A = 0$, $B = 1$, Parabola
   - Right
2. $A = 15$, $B = 2$, Hyperbola
   - Vertical
3. $A = 2$, $B = 2$, Circle
   - None
4. $A = 11$, $B = 7$, Ellipse
   - Vertical
5. Ellipse
6. Ellipse
7. Circle
8. Circle
9. Hyperbola
## Lesson 6

### Learning Task # 1
1. **Arithmetic, Geometric, Harmonic, Fibonacci**

#### Arithmetic Sequence

\[ a_n = a_1 + (n - 1)d \]

#### Geometric Sequence

\[ a_n = a_1 \cdot r^{n-1} \]

### Learning Task # 2
1. 25,000
2. 192
3. Level 8 & 192

### Learning Task # 3
1. Like
2. Like
3. Unlike
4. Like
5. Unlike
6. Unlike
7. Like
8. Unlike
9. Unlike
10. Like

### Learning Task # 4

<table>
<thead>
<tr>
<th>Sequence</th>
<th>Sum (S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>{3, 10, 29, 66, 127}</td>
<td>235</td>
</tr>
<tr>
<td>{-2, 16, 44, 82, 130}</td>
<td>-270</td>
</tr>
<tr>
<td>{-13, -1, -37, 71, -253}</td>
<td>-233</td>
</tr>
<tr>
<td>{1, 3, 6, 10, }</td>
<td>35</td>
</tr>
<tr>
<td>{2, -1, -4, -7, -10}</td>
<td>-20</td>
</tr>
</tbody>
</table>

### Learning Task # 5
1. 30
2. 100
3. 1,319,026
4. 1,79,173,167
5. 47

## Lesson 7

### Learning Task # 1
1. 11, 325
2. 2,500
3. 187,026
4. 1,79,173,167

### Learning Task # 2
1. Like
2. Unlike
3. Like
4. Unlike
5. Like
6. Unlike
7. Like
8. Unlike
9. Like
10. Unlike
Learning Task # 3
1. 30
2. 35
3. 410
4. 899
5. 1,070

Learning Task # 4
1. \( \frac{3}{176} \) and \( \frac{3}{11} \)
2. \( 3k - 2 \) and \( 2 \phi \)
3. \( 10 \sum_{k=1}^{\phi} \frac{3}{(k+1)} \) and \( 10 \sum_{k=1}^{\phi} \frac{3}{(k+1)} \)
4. \( 2 \sum_{k=1}^{\phi} \frac{1}{k+1} - 1 \) and \( 2.10 \)
5. \( \sum_{k=1}^{\phi} 5k \) and \( 2.75 \)

Learning Task # 5
1. \( 10,070 \) and \( 290 \)
2. \( 1,994 \) and \( 191 \)
References


Department of Education. Pre-Calculus learners’ materials. Sunshine Interlinks Publishing House, Inc.


Para sa mga katanungan o puna, sumulat o tumawag sa:

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